Acceleration compensation for gravity sense using an accelerometer in an aerodynamically stable UAV

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Abstract— **This paper considers the problem of compensating for vehicular accelerations in an inertial sensor, in order to obtain a sense of the gravitational field. The gravity sense can then be used to estimate the relative attitude of the sensor with respect to the field. However, the accelerometer in inertial measurement units (IMUs) measures the sum of the inertial acceleration and the gravitational field, and the measurement cannot be directly decomposed into the two components. Separating the gravitational component out is therefore crucial to the use of IMUs for attitude estimation or determination. The separation has typically been accomplished by making a steady turn assumption in unmanned aerial vehicles (UAVs). This paper introduces a new assumption that the aerodynamic forces are nearly constant in the body frame. This assumption leads to significant improvement in estimator performance during perturbations from a nominal motion, when restricted to UAVs that possess inherently stable aerodynamics. The stability properties of the compensator are analyzed to prove that the compensator retains the property of asymptotic stability under a steady turn assumption, and that it performs better despite not being asymptotically stable when the assumption is withdrawn. The resulting improvement in performance is demonstrated both in simulations as well as experiments.**

I. Introduction

It may be shown by a simple dimensional analysis [1] that the time scale of vehicle dynamics varies as the inverse of the square root of the length scale ($T \propto 1/\sqrt{L}$) for fixed-wing airplanes in incompressible flow under a constant external gravitational field. Control of small unmanned aerial vehicles (UAVs) therefore demands faster response times from the state estimator. This provides the motivation for the design of a low-cost, low-weight, low-latency attitude estimator.

Attitude estimation in many unmanned vehicle applications use micro electro-mechanical (MEMS) inertial sensors such as gyroscopes and accelerometers. This is especially true with small UAVs, which are severely constrained with respect to payload mass and size and cannot afford the heavier purely mechanical sensors or vision based sensors. However, the smaller and lighter inertial measurement units (IMUs) are associated with greater non-idealities such as noise, bias, and crosstalk.

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With regards to inertial accelerometers, their usage as a gravity sense is hampered by the fact that accelerometer measurements contain not only information of the gravity field, but also vehicular accelerations. It is well known that an inertial sensor cannot distinguish between accelerations and motion that differs from free-fall in gravity (the principle of mass equivalence). The usage of the sensor to detect gravity in attitude estimation is therefore always accompanied by the associated problem of compensating for vehicular accelerations.

With the development and proliferation of electronic IMUs in recent years, acceleration compensation has been the focus of active research. In [2], [3], the authors compensate for centripetal acceleration, and also include a dynamic model for the angle of attack in order to obtain body-referred components of the airspeed to compute the centripetal acceleration. The dynamic model for the angle of attack is assumed to be obtained from wind-tunnel tests, or flight tests. Moreover, perturbations from the nominal motion on account of highly dynamic motion, or external disturbances, are not considered. In [4], and [5] the accelerometer measurement is progressively ignored when the measured magnitude of acceleration is different from the expected constant acceleration due to gravity of 9.81m/s^2. In [6], the authors propose a Kalman filter for estimating the acceleration and a switching method depending upon the magnitude of the accelerometer measurement. In [7], GPS sensors are used to compute translational acceleration which can then be subtracted from the accelerometer measurement in order to obtain gravity sense. However, this method is not attractive for small UAVs because of the weight and power overheads, besides the avoidable dependency upon an external man-made GPS satellite infrastructure. In [8], vision based sensors are used to supplement IMU based attitude estimation. The solution has applications currently limited to large vehicles on account of size and weight considerations, but with advances in vision and imaging technologies, vision aided attitude estimation may soon become practically feasible even in small UAVs.

In this paper, we compensate for the centripetal acceleration as done in [2], and additionally impose the assumption that the aerodynamic forces are smooth functions of the aerodynamic state, which is specified relative to the body-fixed frame. Since the UAV's aerodynamic state varies continuously in time, so do the forces, and thus the body-frame acceleration is nearly constant across small time steps. Moreover if the UAV's natural dynamics are stable (*resp.* title!), then the changes in the aerodynamic state are small and the deviations in the body-frame acceleration are primarly in order to revert to the stable nominal motion. This new assumption is useful when a UAV experiences significant stabilizing accelerations, which could be a consequence of tracking a desired motion trajectory that is highly dynamic, or a consequence of external disturbances such as wind gusts.

An obvious limitation of the proposed method is that it is not applicable when the aerodynamic forces are discontinuous with respect to state, and hence with respect to time. This may occur, for example, if the change in state is associated with a change in the aerodynamical regime, leading to separation, stall, or other such pathological conditions. For typical flight conditions, the occurence of such phenomena is rare. In particular, it may be noted that this assumption on the flight regime is strictly weaker than the assumption that the airplane is executing a steady turn, which imposes not just inequality constraints on the aerodynamic state variables, but rather equality constraints on them. The proposed compensator would therefore be a forward step in designing effective acceleration compensation for inertial attitude estimation.

A brief outline of this paper is as follows: we briefly establish the notation and terminology, and present a concise mathematical statement of the problem of acceleration compensation in the next section. Section III then describes the proposed solution to the problem, whose stability properties are also analyzed in the next section IV. Finally, we verify the compensator's performance in simulations in section V, and also by way of experiments in section VI.

II. Notation, definitions, and problem statement

We consider the output of a typical IMU, such as the MPU9250 [9], which contains three-axis accelerometer and three-axis gyroscope readings. The IMU is assumed to be strapped down rigidly to the vehicle whose attitude is to be estimated. The body's attitude, and hence the IMU's attitude, with respect to a ground-fixed North-East-Down (NED) frame may be represented using an orthogonal matrix C. The NED frame shall be considered to be inertial, which is a reasonable approximation for the dynamics of small UAVs. Then the body-frame components g of the gravitational acceleration are given in terms of the NED-frame (assumed inertial) components g_N by the transformation equation:

$$g = C^T g_N. \tag{1}$$

Let $\mathbf{f}$ be the vector sum of all the forces acting at the boundary of the body, which, for a UAV, would be the total aerodynamic force, divided by the mass. This specific force would be balanced by the vehicle's acceleration $\mathbf{a}$, and the gravity $\mathbf{g}$ according to the equation

$$\mathbf{a} = \mathbf{f} + \mathbf{g}.$$

In terms of the body-frame components a of $\mathbf{a}$ and f of $\mathbf{f}$, and using (1), we have

$$a = f + g = f + C^T g_N. \tag{2}$$

Let us further decompose $\mathbf{a}$ as $\mathbf{a} = \boldsymbol{\omega} \times \mathbf{V} + \mathbf{r}$, where $\boldsymbol{\omega}$, $\mathbf{V}$, and $\mathbf{r}$ are the angular velocity, translational velocity, and the non-centripetal acceleration of the body respectively. This decomposition is motivated by the assumption that the UAV reverts to a stable steady turn in the absence of control inputs and external disturbances. Let the vectors $\mathbf{r}$, $\boldsymbol{\omega}$, and $\mathbf{V}$ have components r, ω, and V along the body-fixed axes. The force balance equation (2) for the vehicle's translational dynamics may then be written as:

$$r + \omega \times V = f + C^T g_N. \tag{3}$$

In the above equation, g_N is a known constant, and it is possible to inertially sense ω, and f. We shall also assume that V is measured using an airspeed sensor. Then, the remaining unknowns in the above equation are the non-centripetal acceleration r and the attitude matrix C. When the UAV is executing a steady turn or steady level flight (which is a degenerate steady turn), r is also zero, and (3) reduces to

$$\omega \times V = f + C^T g_N \qquad \text{(when } r = 0\text{)}. \tag{4}$$

Since C is orthogonal, (4) has a scalar degree of redundancy, and we can deduce two degrees of freedom of the attitude using (4). Together with the attitude matrix dynamics

$$\dot{C} = C\Omega, \tag{5}$$

where $\Omega = [\omega\times]$ is the angular velocity cross product matrix, we have the system set up for complementarily filtering and estimating C as $\hat{C}$. This is the most frequent method adopted for acceleration compensation in inertial attitude estimation (the "uncompensated" equation is $f/\|f\| + C^T g_N/\|g_N\| = 0$).

We would like to now rectify the deficiency in approximating (3) using (4). For an aerodynamically stable vehicle, deviations from equation (4) would occur on account of control input or external disturbances. While the non-centripetal acceleration r in (3) may be small in magnitude in comparison with g_N during trimmed and undisturbed flight, it can easily approach comparative orders of magnitude in small UAVs, during entrance into or exit from a steady turn, or on account of external disturbances. These deviations are especially significant for aerobatic flight and at low airspeeds. We shall therefore try to include r in equation (3) and hence obtain an accurate estimate for C across a wider range of flight conditions. Our problem may therefore be stated with reference to (3) as *estimating the body-frame components of gravity $g = C^T g_N$ in the presence of an unknown acceleration a, in terms of body-frame measurements of angular velocity ω, aerodynamic force per unit mass f, and velocity V.*

III. Acceleration compensation

The total acceleration a, and the gravity in the body-frame $C^T g_N$, in (2), cannot be directly measured. There are two approaches to designing estimators $\hat{a}$ and $\hat{C}$ for the acceleration a and rotation matrix C: one in the inertial reference frame, another in the body-fixed frame. Both these approaches are equivalent and lead to the same stability results and performance, though the latter approach is more elegant and simple to follow. While the first approach is theoretically equivalent to the second approach (to be discussed below), it is associated with the practical drawback that the angular velocity needs to be estimated in the inertial reference frame as $\hat{\Omega}_N = \hat{C}\Omega\hat{C}^T$. This transformation is inefficient since we would like to ultimately estimate the body-frame components of the acceleration a. We therefore formulate the solution in the body-frame so as to avoid the inefficiency. The second approach incorporates the effects of body rotation directly in the estimation of a, and leads to a more elegant form of estimator dynamics.

The dynamics of $\hat{a}$ are inspired by those of the unknown quantity a given by the following expression:

$$\begin{aligned} a &= C^T g_N + f \\ \Rightarrow \dot{a} &= -\Omega C^T g_N + \dot{f} \\ &= \Omega(f - a) + \dot{f}, \end{aligned} \tag{6}$$

where the expression on the right hand side may be approximated as $\Omega(f-a)$ if $\dot{f} \approx 0$ during steady turn. Note that the time derivative $\dot{f}$ is well defined in the normal flight regime of smooth variations in the aerodynamic state. We therefore estimate a as $\hat{a}$ according to:

$$\dot{\hat{a}} = \Omega(f - \hat{a}) + W\left(\Omega V - \hat{a}\right), \tag{7}$$

where W is the rate at which the UAV dynamics revert naturally to a steady turn. The first term on the right hand side of (7) signifies the near constancy of the aerodynamic force components f in the body-frame, while the second term signifies the gradual reversion of the dynamics to the stable nominal motion (stable nominal motion being a steady turn). Ideally, the rate matrix W would satisfy the below equation:

$$\dot{f} = -Wr, \tag{8}$$

which indicates that a non-zero value for the unsteady acceleration r generates an opposing aerodynamic force that tends to stabilize the translational motion of the vehicle. This physical principle manifests as negative values for stability derivatives such as $C_{Y,\beta} = \partial f_2/\partial\beta$, where f_2 and β are the sideforce and sideslip angle respectively. Similar stabilizing principles hold for accelerations along the x and z directions in the body-frame: the dominant effect of a downward velocity is the generation of higher lift, and an increasing forward velocity produces increasing drag.

In practical applications, it is not possible to know the exact value of W, other than that it is positive definite for stable translational dynamics. Furthermore, $\dot{f}$ also depends upon angular accelerations. For example, the time-derivative of the sideforce $\dot{f}_2$ depends upon $C_{Y,p}\dot{p}$, where $C_{Y,p}$ is its stability derivative with respect to the roll rate p. The aerodynamic coefficients, $C_{Y,\beta}$, $C_{Y,p}$ *etc*, are themselves functions of the aerodynamic state and uncertain. As we shall see, such practical considerations limit the performance of the compensator to bounded stability rather than asymptotic stability. A more realistic description of $\dot{f}$ would therefore be:

$$\dot{f} = -Wr - v, \tag{9}$$

where v is the residual term in $\dot{f}$ after accounting for r.

Once we have an estimate $\hat{a}$ for the total acceleration a, the attitude may subsequently be estimated from $\hat{a}$ using a complementary filter [2] upon the redundant system of equations:

$$\dot{\hat{C}} = \hat{C}\Omega, \tag{10}$$

$$\hat{C}^T g_N = \hat{a} - f. \tag{11}$$

Equations (7), (10), and (11) present a coupled set of equations to estimate the total vehicle acceleration a as $\hat{a}$, and the attitude matrix C as $\hat{C}$ (or equivalently, the attitude quaternion $\hat{\bar{q}}$).

IV. Stability properties of the compensator

Let us now analyze the stability properties of the acceleration compensator prescribed by (7), (10), and (11) in the previous section. In order to do this, we define an acceleration estimation error signal $e = \hat{a} - a$. Using (3) ($a = \Omega V + r = C^T g_N + f$), (6), (7), and (9), we obtain:

$$\begin{aligned} \dot{e} &= \dot{\hat{a}} - \dot{a} = \Omega(f - \hat{a}) + W(\Omega V - \hat{a}) + \Omega(a - f) - \dot{f} \\ &= \Omega(a - \hat{a}) + W(a - r - \hat{a}) - \dot{f} \\ \Rightarrow &\left(\frac{d}{dt} + W + \Omega\right) e = -Wr - \dot{f}. \end{aligned} \tag{12}$$

By referring to (8), we see that in the ideal case, the error dynamics are asymptotically stable. This can be proven by considering the following Lyapunov function:

$$V_L = \frac{1}{2} e^T e, \tag{13}$$

$$\begin{aligned} \Rightarrow \dot{V}_L &= e^T \dot{e} = e^T \left(-(W + \Omega)e - Wr - \dot{f}\right) \\ &= -e^T W e \qquad (W \text{ exactly known}), \end{aligned} \tag{14}$$

where we have used the facts that Ω is skew-symmetric, and that $\dot{f} + Wr = 0$ for an ideal choice of W. In case of a non-ideal (but still positive definite) description $\hat{W}$ of W, and a non-zero residual v in (9), we would have:

$$\begin{aligned} \dot{\hat{a}} &= \Omega(f - \hat{a}) + \hat{W}(\Omega V - \hat{a}), \\ \Rightarrow &\left(\frac{d}{dt} + \hat{W} + \Omega\right) e = -\hat{W}r - \dot{f} = -\tilde{W}r + v, \\ \dot{V}_L &= -e^T \hat{W} e - e^T \tilde{W} r + e^T v, \end{aligned} \tag{15}$$

where $\tilde{W} = \hat{W} - W$ is the residual error in estimating W. Under steady turn conditions, we have $r = 0$ and $v = 0$, thus immediately yielding asymptotic stability, as in equations (13) and (14), with $\hat{W}$ replacing W.

Since $\hat{W}$ in (15) is positive definite by choice, its symmetric part, $(\hat{W} + \hat{W}^T)/2$, can be spectrally decomposed as $Q\Lambda^2 Q^T$, where Q is orthogonal, and Λ is an invertible diagonal matrix. Let R denote the invertible matrix $Q\Lambda$. This allows us to complete the square and express (15) as

$$\dot{V}_L = -e^T R R^T e - e^T \tilde{W} r + e^T v \tag{16}$$

$$= -\left\| R^T e + \frac{R^{-1}(\tilde{W}r - v)}{2} \right\|^2 + \frac{\|R^{-1}(\tilde{W}r - v)\|^2}{4}.$$

In the more general case with $\dot{f} \neq 0$, (16) shows that we can only guarantee bounded stability that $\|e\| \leq (\|(\tilde{W}\|\|r\| + \|v\|)/\lambda_{min}$, where we have assumed an appropriate induced matrix norm for $\|\tilde{W}\|$, and λ_{min} is the smallest eigenvalue of $\hat{W}$.

It is tempting to attempt to adaptively, or otherwise, estimate the elements in the matrix W and vector v. However, it must be borne in mind that the force balance equation (3) introduces three scalar unknowns r in its three scalar equations. Without an accurate description of the translational kinetics (besides the kinematics), the system is under-determined at each single time point. Even for a time-sequence of measurements, the system still remains under-determined as long as the elements of W and v themselves are state-dependent and keep varying with time in an unknown fashion. Thus, there are no asymptotically stable solutions to acceleration compensation in the absence of accurate knowledge of translational kinetics (the matrices W and v).

The error dynamics in (12) may be compared to those obtained by not accounting for the residual acceleration r, but just using the steady turn assumption:

$$\begin{aligned} \hat{a} &= \Omega V, \\ \Rightarrow e &= \hat{a} - a = \Omega V - a = -r. \end{aligned} \tag{17}$$

It may be seen at once by comparing (12) and (17) that (17) is the limit of (12) when $\hat{W} \to \infty$, *i.e.,* when the estimator tries to quickly revert $\hat{a}$ to ΩV. At the other extreme, when $\hat{W} \to 0$, we see that $\dot{e} = -\Omega e - \dot{f} \approx -\Omega e$, and for a skew-symmetric matrix, that implies that the magnitude of the error remains constant. For values of $\hat{W}$ in between these two extremes, it is possible to obtain a value that approaches W and leads to optimum estimator performance.

V. Simulation Results

In this section, the acceleration compensator of the previous section is evaluated in Matlab simulations for a tuned rate matrix $\hat{W}$. The elements of $\hat{W}$ may also be derived analytically in terms of the stability derivatives, but we shall not assume that the UAV's aerodynamic model is exactly known and only use approximate values for the model parameters. The elements of $\hat{W}$ are therefore derived by tuning for optimum performance. In practice, this may involve experimental trial and error using system identification tools. For our simulations, we choose a diagonal matrix $\hat{W} = \mathrm{diag}\,[1\ 1\ 10]$.

In the following simulations, we use a model for a fixed-wing UAV [10] flying at an airspeed of 10m/s while executing waypoint navigation. In a first set of simulations (figures 1, 2, and 3), we demonstrate the improvement in estimator performance when the UAV goes through unsteady accelerations while entering or exiting a turn. In the second set of simulations (figures 4, 5, and 6), we demonstrate the performance improvement in the presence of external disturbances in the form of sinusoidal wind.

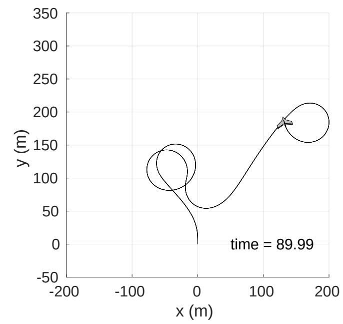


Fig. 1: Ground track of a UAV performing waypoint navigation. The UAV is flying at 10m/s, first to a waypoint at (-50, 100), and subsequently to (150, 200). The waypoint switch occurs at time $t = 45s$.

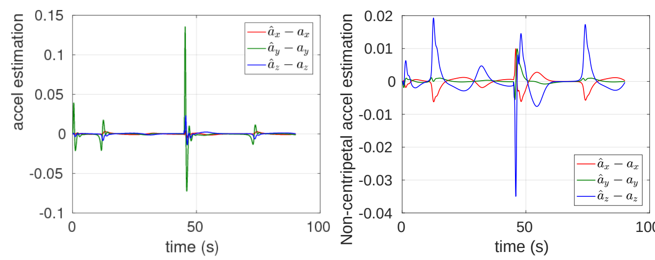


Fig. 2: Estimation of UAV total acceleration a in equation (3). Left: Estimation error by assuming a steady turn $\hat{a} = \Omega V$. Right: Estimation error by assuming constancy of aerodynamic force f in body-frame using equation (7), and gradual approach to steady motion.

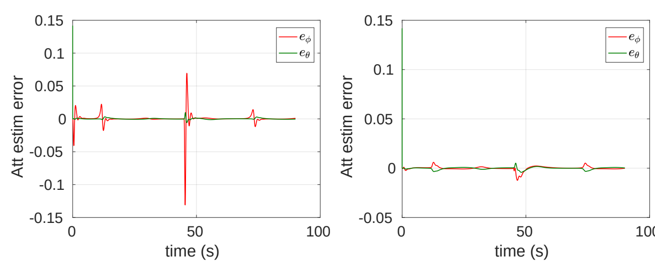


Fig. 3: Estimation of UAV attitude using equations (10) and (11). The attitude matrix has been expressed in terms of the roll and pitch angles for ease of interpretation. Left: Estimation error by assuming a steady turn $\hat{a} = \Omega V$. Right: Estimation error by assuming constancy of aerodynamic force f in body-frame using equation (7), and gradual approach to steady motion.

The test scenario for the first set of simulations is designed to demonstrate the superior acceleration compensation when the UAV executes unsteady manoeuvres. It consists of the UAV flying to two different waypoints as shown in the ground-track in figure 1. The airspeed of the UAV is 10m/s, the initial position is at $x = y = 0$, and there is no ambient wind. In this scenario, the UAV is mostly in steady motion, except when it begins the approach towards the first waypoint at $t = 0$, reaches the waypoint at $t \approx 15s$, switches to a new waypoint at $t = 45s$, and reaches the second waypoint at $t \approx 70s$. It may be seen in figures 2, and 3 that the proposed compensator is asymptotically stable during the steady phases of motion, while it results in better acceleration estimation and consequently better attitude estimation during the unsteady phases. The estimation error of $\hat{a}$ improves four-fold, while the attitude estimation error improves nearly eight-fold.

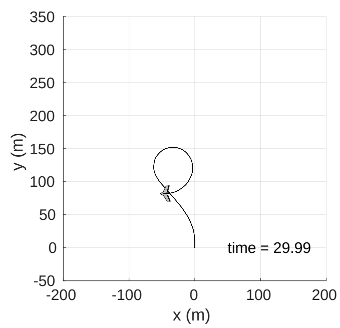


Fig. 4: Ground track of a UAV performing waypoint navigation. The UAV is flying at 10m/s to a waypoint at (-50, 100). Once it reaches the waypoint at $t \approx 15s$, it loiters around it by executing turns. All the while, the UAV experiences a sinusoidal wind with peak-to-peak amplitude of 2m/s. It may be noted that the frequency, phase, and amplitude of the disturbance are not relevant to the simulation: they are chosen arbitrarily to demonstrate the relative superiority of the proposed acceleration compensation.

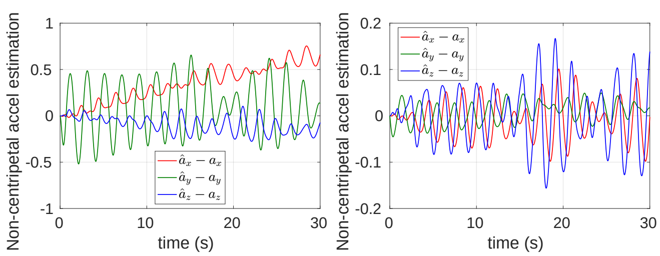


Fig. 5: Estimation of UAV total acceleration a in equation (3). Left: Estimation error by assuming a steady turn $\hat{a} = \Omega V$. Right: Estimation error by assuming constancy of aerodynamic force f in body-frame using equation (7), and gradual approach to steady motion.

In the next set of simulations, the test scenario is a small UAV flying to a waypoint as shown in the ground-track in figure 4 while experiencing ambient wind that has a peak-to-peak amplitude equal to 20% of the UAV's airspeed. The airspeed of the UAV is 10m/s and the ambient wind therefore has a peak-to-peak amplitude of 2m/s. In this scenario, the UAV is almost always in unsteady motion on account of the unsteady disturbance. Moreover, the non-centripetal acceleration reaches magnitudes equal to 50% of g_N even at the small external wind amplitude. As the UAV turns around the waypoint, the constant direction of the wind in

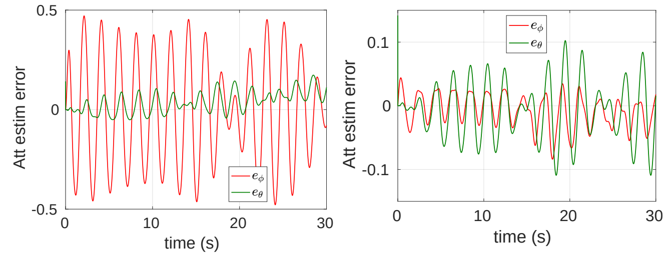


Fig. 6: Estimation of UAV attitude using equations (10) and (11). The attitude matrix has been expressed in terms of the roll and pitch angles for ease of interpretation. Left: Estimation error by assuming a steady turn $\hat{a} = \Omega V$. Right: Estimation error by assuming constancy of aerodynamic force f in body-frame using equation (7), and gradual approach to steady motion.

the inertial reference frame becomes time-varying in the body-frame. It may be seen in figures 5, and 6 that the proposed compensator leads to better acceleration estimation and consequently better attitude estimation. The estimation error of $\hat{a}$ improves five-fold, while the attitude estimation error improves nearly four-fold.

Thus, we see that the proposed acceleration compensation improves estimator performance both in the presence and absence of external disturbances. Moreover, the compensator does not need a precise knowledge of the vehicle's aerodynamic model, or the rate matrix W in (8). The compensator can only provide bounded performance, but the bounds are an order better than those obtainable using compensators reported previously.

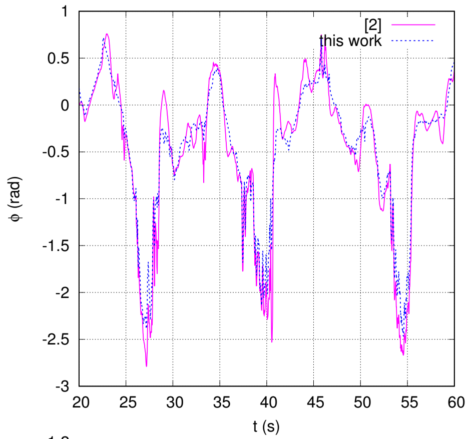
VI. Experimental verification

We finally verify the compensator and attitude estimator performance on a real UAV. The UAV is a 0.94m span fixed-delta-wing airplane. While the fuselage was commercially purchased, the hardware and avionics were assembled in-house including the design and development of the autopilot [11]. The autopilot has the MPU9250 installed to provide the inertial measurements of the angular velocity Ω and aerodynamic acceleration f. A nominal aerodynamic model of the UAV has also been reported in [10] for trimmed flight.

Figure 7 shows a comparison of the UAV attitude as obtained using the compensator presented in this paper against that presented in [2]. It can be seen that the estimator presented in this paper yields smoother results without smaller discontinuities (*e.g.*, at time $t \approx 40s$). Moreover, the assumption that the total acceleration is the same as that due to a steady turn results in significantly larger errors in comparison with the compensator presented in this paper.

VII. Conclusion

We have thus presented a new technique for acceleration compensation that improves the transient performance of an inertial sensor measuring the gravity. The technique incoporates the near continuity of aerodynamic forces as it rotates to change its attitude. This new information leads



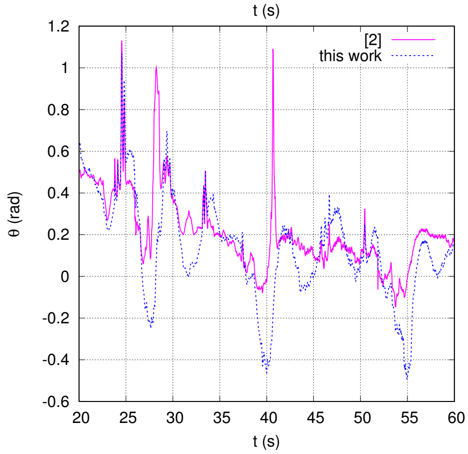


Fig. 7: Roll and pitch estimation using the accelerometer as a gravity sensor on a real IMU, the MPU9250, mounted on a UAV. In both plots, the solid magenta curve is obtained by assuming that the only acceleration is due to a steady turn, while the dashed blue curve includes the transport term assuming aerodynamic forces rotate with body. Top: Roll angle estimation. Bottom: Pitch angle estimation.

to better compensation of acceleration, and hence a more accurate sense of gravity, which in turn leads to better attitude estimation. The stability properties of the compensator have been analyzed and the performance improvement has been demonstrated in simulations as well as experiments.

A focus for future research involves deriving a reasonably accurate estimation of the rate matrix W in (7). The elements of this matrix are expected to be functions of the aerodynamic coefficients of the UAV. Another ongoing experiment is to compare the compensator presented in this paper against a "gold standard" benchmark estimator such as a vision based estimator.

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