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STUDIES OF TWO-DIMENSIONAL VORTEX STREETS

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Abstract

Periodic vortex streets, often considered as a model for the organized structures observed in the wake of two-dimensional (2D) bluff bodies, are revisited. After the generation of the shear layers the formation of the vortices is mainly an inviscid process. An intrinsic scaling for the formation of vortex streets is found based on the invariants of motion for 2D inviscid flows; namely the kinetic energy, impulse, circulation and the translational velocity of the vortex system. We show that in the frame of reference of the invariants of motion the intrinsic shedding Strouhal number St defined based on the invariants of motion and the aspect ratio of the vortex street κ (the ratio of the lateral distance to the streamwise distance between the centers of vortices) are literally the same. The formation of the vortex street is a manifestation of Kelvin's variational principle. Using the computational results from Saffman and Schatzman[1] for the inviscid vortex streets of vortex patches we estimate values of the nondimensional energy and circulation of the vortex system for a wide class of vortex patches. A relaxational explanation of the vortex shedding is also offered. In this picture the bluff body is considered as a source of providing the system with invariants of motion. After the formation of the shear layers the system will relax toward its final equilibrium state where the formation of a vortex street is mandated by the invariants of motion for the two-dimensional Euler equations. Similar to the vortex ring pinch-off process it is speculated that the characteristics of the vortex system can be modified by modification in the rate of generation of invariants of motion. The two main methods for modifying a vortex system are the dynamical changing of the speed and the lateral spacing of the generated shear layers during the formation of circulation regions. This is often achieved by periodic streamwise and lateral oscillations of the bluff body.

1 Introduction

The wake of the two-dimensional bluff bodies in a uniform stream usually shape into a regular pattern of two parallel staggered rows of vortices for a large range of Reynolds numbers; see for example Williamson[2] and the references in there. Vortices which are formed at the two points of separation on the bluff body, are seen to be shed off regularly in an alternating fashion, as shown in figure 1, in which U_∞ is the uniform velocity of the approaching stream, and U_t is the translational velocity of the vortex street with respect to the free stream. It is apparent that viscosity does not play an essential role in the formation of the vortex street once the shear layers have been developed. This is supported by investigations on the inviscid evolution of infinite parallel vortex layers with small periodic disturbances (see Aref and Sigga[3] and the references in there); the Karman vortex street seems to be the generic result in such inviscid calculations. Furthermore, it has long been experimentally observed that the Strouhal number does not change significantly for a wide range of Reynolds numbers (*e.g.* see Roshko[4]).

The double trail of vortices formed alternately on both sides of a cylinder was modeled by von Karman[5] as a regular pattern of point vortices in inviscid flows. von Karman not only analyzed the stability of a system of vortex streets, but also established a theoretical link between the vortex street configuration and the drag coefficient on the body. He studied both the symmetric and antisymmetric configurations of a double row of point vortices. He showed that the only configuration that does not exhibit linear instability is for an antisymmetric configuration (see figure 1) with a vortex spacing of $\kappa_c = \sinh^{-1} 1/\pi \approx 0.28$. See Lamb[6] and Saffman[7] for a review of the stability analysis including the effect of finite core size and three dimensional instability. It should be noted that these analyses do not address the issue of formation of vorticity on the body. However, they suggest that if such vortex streets are formed they should be observed for a

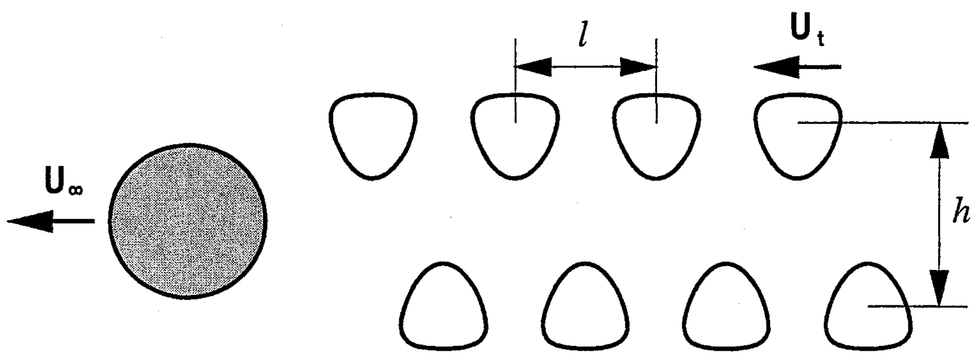

Figure 1: Vortex street behind a bluff body. Note that for an observer traveling with the vortices the velocity at infinity is U_t, and the cylinder moves away from the vortices at a relative velocity of $U_\infty - U_t$.

reasonable distances downstream.

Vortex streets are the two-dimensional (2D) counterpart of an array of axisymmetric vortex rings. Our results here is a natural extension of the relaxational idea developed in Mohseni,[8] Mohseni and Gharib,[9] and Mohseni *et al.*[10] A brief review of the vortex ring pinch-off process in axisymmetric flows is presented in section 2.

In this paper an intrinsic scaling for a regular vortex street based on the main invariants of motion of the 2D Euler equations is introduced. The relevant quantities are then nondimensionalized based on these scales. This scaling fits well with Kelvin's variational principle. Saffman and Schatzman[1,11] and Schatzman[12] studied the steadily translating solutions of vortex streets consisting of vortex patches (vortices of finite area and uniform vorticity) as the simplest extension of von Karman's point vortex model. Saffman and Schatzman[11] mathematically addressed the question of whether constant vorticity structures of finite extent can stabilize a vortex street. They found numerical solutions of the Euler equation for their model and calculated the properties of the street, that we used in the present study. In passing we note that Meiron *et al.*[13] concluded that the modeling of a vortex street by regions of constant vorticity did not fundamentally alter the stability results of Karman's point vortex model; at least for moderate values of the area of the individual vortices, there is only a single value of κ for which the street is stable. This behavior is a consequence of the Hamiltonian nature of the Euler equations and its symmetries[14] and it should hold for any inviscid model of the street that retains the back to fore symmetry of the basic flow. Jimenez[14] argues that the viscous effects might be responsible for the persistence of a natural vortex street observed in experiments.

This paper is organized as follows. In the next section we review the relaxational approach in the vortex ring pinch-off process. This will motivate the application of the same approach to the Karman vortex street. Equivalence of the intrinsic shedding Strouhal number St in the frame of reference of invariants of motion and Karman's aspect ratio κ was studied in section 3. Kelvin's variational results were also addressed in section 3. Nondimensionalization of the Karman vortex street based on the invariants of motion is performed in section 4. In section 5 the numerical computations of Saffman and Schatzman[1] were used to estimate the nondimensional energy and impulse of the vortex system for various aspect ratios κ. An explanation for the vortex shedding behind bluff bodies based on the relaxational model is discussed in section 6. Finally we summarize our results in section 7.

2 Universality in vortex ring pinch-off process

Our main motivation for the present investigation comes from the universal formation number of vortex ring pinch-off process observed in experiments by Gharib *et al.*[15] Mohseni & Gharib[9] offered a relaxational model for the vortex ring pinch-off process. Numerical simulations of the Navier-Stokes equations were performed by Mohseni *et al.*[10] where they verified the modeling assumptions in Mohseni & Gharib.[9] In the laboratory, vortex rings can be generated by the motion of a piston pushing a column of fluid through an orifice or nozzle. The boundary layer at the edge of the orifice or nozzle will separate and roll up into a vortex ring. We think that since the formation of vortex rings involves strong mixing of the generated shear layer with the ambient fluid (the same applies to the formation of vortices in two-dimensional flows), the ergodicity requirement of statistical equilibrium theories has a chance to be satisfied. The experiments of Gharib *et al.*[15] have shown that for large piston stroke versus diameter ratios (L/D), the generated flow field consists of a leading vortex ring followed by a trailing jet. The vorticity field of the formed leading vortex ring is disconnected from that of the trailing jet at a critical value of L/D (dubbed the "formation number"), at which time the vortex ring attains a maximum circulation. The formation number was in the range 3.6 to 4.5 for a variety of exit diameters, exit plane geometries, and non-impulsive piston velocities. An explanation for this phenomenon was given based on Kelvin's variational principle. It was both experimentally[15] and analytically[9] observed that the limiting stroke L/D occurs when the generating apparatus is no longer able to deliver energy, circulation and impulse at a rate comparable with the requirement that a steadily translating vortex ring has maximum energy with respect to kinematically allowable perturbations. Recently Mohseni[8] argued that the energy extremization in Kelvin's variational principle has a close connection with the entropy maximization in statistical equilibrium theories. Numerical evidence for a relaxation process in axisymmetric flows to an equilibrium state has already been provided by Mohseni *et al.*[10] in a direct numerical simulation of the vortex ring pinch-off process.

Inspired by these observations we offered a relaxational (statistical) approach to the vortex ring pinch-off process.[9,10] This is an alternative explanation of the vortex ring pinch-off process, based on a mixing entropy maximization, besides the energy extremization approach in Kelvin's variational principle. From this point of view, any vortex ring generator can be viewed as a tool for initializing an axisymmetric flow with a particular rate of the generation of invariants of motion. Each vortex ring generator has a specific rate for feeding the flow with the kinetic energy, impulse, circulation, *etc.* In this picture, at small strokes (small L/D) one will find that all of the initial vorticity density will coalesce into a steadily translating vortex ring. As the stroke length increases the size, strength, and the translational velocity of the resulting vortex ring increase. This process persists until a critical formation number is reached, when the vortex generator is not able to provide invariants of motion compatible with a single translating vortex ring. Equivalently, beyond the critical formation number a single vortex ring at equilibrium (steadily translating) that maximizes the mixing entropy for given energy, impulse and circulation is not possible. In this case the leading vortex ring will pinch-off from the trailing jet and will relax to a translating vortex ring with the translational velocity U_t dictated in the maximum entropy principle. For very large strokes (greater than twice the critical formation number) successive vortex rings will pinch-off from the the trailing jet. This scenario was verified in the numerical simulations of the vortex ring pinch-off process in Mohseni *et al.*[10] The general observation in these simulations was that the main invariants of motion in the pinch-off process are the kinetic energy, circulation and impulse. The higher enstrophy densities *did not* play a significant role as long as the Reynolds number was relatively high.

In the next sections we utilize the same approach to study the Karman vortex street behind bluff bodies. We view the vortex street formation as a relaxational process where the generated shear layers relaxes into coherent vortical structures while maximizing a mixing entropy or equivalently extremizing the energy of the system (Kelvin's variational principle). Statistical equilibrium of a vorticity distribution in 2-D flows was studied by Miller,[16] Miller *et al.*,[17] and Robert and Sommeria.[18]

3 Frame of reference of invariants of motion

Apart from the vorticity generation process, the formation of vortex streets is mainly an inviscid process. The invariants of motion for 2-D Euler equations include the energy E, circulation Γ, impulse I, and the generalized enstrophies. However, on any finite resolution the only invariants of motion that survive the mixing process are the linear functionals of vorticity,[8,17] namely E, Γ, and I. The translational velocity U_t of the center of the vorticity field is also an invariant of motion. Note that U_t appears as the Lagrange multiplier for the impulse in the variational formulation underlying the Euler equations. Therefore, the main kinematical invariants of motion in the formation of vortex streets are: E, Γ, I and U_t. Apart from these kinematical invariants the geometrical parameters h and l (see figure 1) are also invariants of motion.

The surviving invariants of motion may be used to find an intrinsic scaling for vortex streets. This is done in the next section. In this section we show that in the frame of reference of the invariants of motion the intrinsic Strouhal number St and the Karman aspect ratio κ are the same. We define the intrinsic Strouhal number based on the separation distance h, shedding period $T = 1/f$, in which f is the shedding frequency measured by an observer moving with the free stream and measuring a translational velocity U_t for the vortices. Therefore

$$St = \frac{fh}{U_t} = \frac{h}{TU_t} \tag{1}$$

Note that All the parameters in defining this intrinsic Strouhal number are invariants of motion. The shedding time is clearly the same as the time required for a vortex in the vortex street to travel one period l. Hence, $T = l/U_t$, and we obtain

$$St = \kappa = \frac{h}{l}. \tag{2}$$

Therefore, for an observer in the frame of reference of invariants of motion the intrinsic Strouhal number St defined in (1) and the aspect ratio $\kappa = h/l$ (reminiscence of the von Karman aspect ratio used in the stability analysis of Karman's point vortex street) are the same. Note that h is twice the distance of the center of a vortex from the wake centerline.

Apart from the measurement of the shedding frequency in the frame of reference of invariants of motion the Strouhal number defined in equation (1) also deviates from that introduced by Roshko[19] in the length scale parameter used, and is different from that introduced by Bearman[20] in the velocity parameter used. Instead of the vortex street width h, Roshko based it on the distance of the two free shear layers separated from the tripping cylinder. For velocity scale, we use the translational speed of the vortex street, while the velocity at the separation point on the cylinder surface was used by Bearman.[20] Although our definition of the Strouhal number is different than the available definitions in the literature, their values are relatively close. Consequently this might explain why for a fixed stationary bluff body the Strouhal number for vortex shedding behind a bluff body is very close to the von Karman's stability aspect ratio, 0.28.

From the point of view of the theory developed in this study the departure of the values of the Strouhal number and the Karman aspect ratio for a stably translating vortex street stems from the fact that the quantities that are used in the literature are mostly not an invariant of motion for 2-D Euler flows. In general the invariants of motion (*e.g.* U_t and h) are not known a priori, while the dimensions of the bluff body D and the free stream velocity U_∞ are often given. This fact makes the definition of the Strouhal number based on easily available scales more desirable and practical.

To connect the intrinsic Strouhal number (1) to any other Strouhal number defined in the literature we assume a general Strouhal number St_H defined for a general frame of reference with a distinctive velocity and length scales, V and H respectively. Therefore,

$$St_H = \frac{f_H H}{V}, \tag{3}$$

where H is the characteristic length of the bluff body and f_H is the shedding frequency measured in this frame of reference. One can easily show that

$$\frac{St_H}{St} = \frac{H}{h}\left(1 - \frac{U_t}{V}\right). \tag{4}$$

where the Doppler effect is invoked to relate the frequency in the two frame of references. This relation shows that to connect the vortex street in the wake to the bluff body one requires to measure two parameters experimentally, namely H/h and U_t/V. This is consistent with the previous observations that any connection between the wake and the vortex shedding of a bluff body requires two free parameters that needs to be measured or calculated independently.

If we assume that in the frame of reference of invariants of motion for the Euler equations $St = \kappa$ is universally invariant for a fixed stationary body (as argued in the next section with a value comparable with Karman's aspect ratio for the stability of Karman's point vortex streets) one realizes that the vortex shedding for various bluff bodies or free stream flow parameters are characterized by the ratios H/h and U_t/V. Therefore the main experimental relations needed to relate a general Strouhal number St_H to the intrinsic Strouhal number St are relations for H/h and U_t/V. For a fixed stationary bluff body, where the relative rates of generation of the main invariants of motion are approximately constant during the formation process, we speculate (see next sections) that the resulting Karman aspect ratio would be close to 0.28. However if by any mechanism* the relative rates of generation of

*This can be achieved by forced lateral or streamwise motion of the cylinder or application of a synthetic jet to the near wake of the bluff body the main invariants of motion are changed during the formation of each vortex region one expects a strong modification in the characteristics of the resulting vortex streets, including strong deviation of the Karman aspect ratio from 0.28. See next sections for more discussion on this issue.

4 NONDIMENSIONALIZATION

We would like to consider vortex shedding behind a bluff body as a relaxation problem similar to the relaxation of axisymmetric flows to a regular array of vortex rings discussed in section 2 and in Mohseni.[8] As discussed in the previous sections the main parameters for the resulting Karman street are E, Γ, I, U_t, h, and l, in which h and l are the lateral and streamwise distance between the center of vortex patches, respectively. Note that the period of vortex shedding in the frame of reference of invariants of motion is related to U_t and l through $l = TU_t$.

It was pointed out by Kelvin[21] (see also Arnold[22]) that for, given vorticity and momentum, steady states of 2D Euler flows correspond to stationary points of the kinetic energy with respect to kinematically allowable isovortical perturbations. When the strength of the vorticity is uniform, the requirement of kinematically allowable perturbations in Kelvin's variational method corresponds to keeping the area constant. Kelvin also remarked that the configuration would be stable if the stationary value were a maximum or minimum, but unstable if it were a minimax. In this case the total area A of the uniform vorticity is also an invariant of motion. Therefore one can expect the functionality

$$\kappa := \frac{h}{l} = f(E, \Gamma, I, U_t, h, A). \tag{5}$$

for the aspect ratio κ. Note that E and I are calculated per unit depth and unit streamwise period. A straight forward dimensional analysis results in the following three nondimensional numbers, namely

$$E_{nd} = \frac{E}{IU_t} \tag{6}$$

$$\Gamma_{nd} = \frac{\Gamma}{(IU_t)^{\frac{1}{2}}} \tag{7}$$

$$A_{nd} = A\left(\frac{\Gamma}{I}\right)^2 = \frac{A}{h^2}. \tag{8}$$

Since in the frame of reference of invariants of motion $St = \kappa$ we can cast the relation (5) as

$$St = \kappa = f(E_{nd}, \Gamma_{nd}, A_{nd}). \tag{9}$$

In the next section we explore the dependence of E_{nd} and Γ_{nd} on the values of $St = \kappa$ and A_{nd} for a wide class of inviscid vortex streets.

5 ESTIMATIONS FOR E_{nd} AND Γ_{nd}

To estimate the behavior of E_{nd} and Γ_{nd} for various κ we use the numerical simulations of the Euler equations performed in Professor Saffman's group at Caltech in the 80's. Saffman & Schatzman[1,11,12,23] studied the Karman street of the two dimensional arrays of vortices of finite area and uniform vorticity. This arrangement is believed to be a better approximation of the vortex streets formed behind cylinders than the point vortices of von Karman. Saffman & Schatzman consider a double staggered array of vortices with their centers h apart and with the wavelength of the periodic vortex array being l.

Here we use the calculations by Saffman & Schatzman[1,23] to estimate values of E_{nd} and Γ_{nd} for various aspect ratios κ. We denote any quantity extracted from Saffman & Schatzman's data by '^'. Note that in Saffman & Schatzman[1,23] all quantities are nondimensionalized by the circulation Γ and the wavelength l. Their energy is calculated per unit length, while in our calculations energy is calculated per unit period l. Saffman & Schatzman[1,23] curve-fitted the results of their numerical calculations for values of κ between 0.1 and 0.8 which are used in the present study.

Relations for the energy and the translational velocity of the curve-fitted results are given by Saffman & Schatzman.[1] The area of a vortex is clearly related to the strength, *i.e.* circulation, of each vortex. The area parameter in Saffman & Schatzman[1,23] is $\alpha = A/l^2$. Therefore $A_{nd} = \alpha/\kappa^2$. The relationship between the area of the vortices and κ, required in the calculation of $\hat{E}$ and $\hat{U}_t$, can be obtained from Schatzman.[12]

Now since $\hat{U}_t$ is only a function of κ and α, we can write

$$\Gamma_{nd} = \Gamma_{nd}(\kappa, \alpha) = \frac{1}{\kappa \hat{U}_t}. \tag{10}$$

Therefore both Γ_{nd} and A_{nd} are at most functions of κ and α. Consequently, the nondimensional relation (9) may be translated to a relation for the nondimensional energy E_{nd} as a function of κ and α. Hence

$$E_{nd} = E_{nd}(\kappa, \alpha) \tag{11}$$

Using Saffman & Schatzman's result[1] the nondimensional energy $E_{nd}(\kappa, \alpha)$ is calculated and shown in

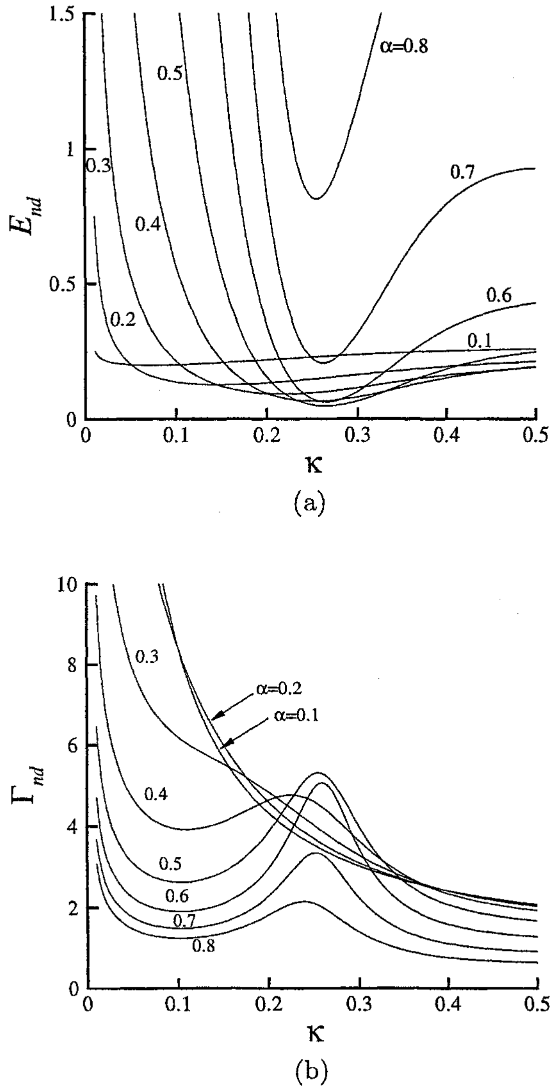

Figure 2: Nondimensional energy (a) and circulation (b) versus κ for various α.

figure 2(a). The corresponding nondimensional circulation is shown in figure 2(b). For some class of vortex streets there is a preferred value of $\kappa = St$ around 0.26. In analogy with the vortex ring pinch-off process we speculate that these vortex streets are generated by a fixed stationary bluff body where the relative rate of generation of invariants of motion are relatively constant during the formation of each vortex. However if these rates are changed dynamically during the formation of each vortex a different equilibrium state with possibly different value of spacing ratio κ will be formed. This idea is explored in the next section.

An objective in the modeling of vortex shedding in the wake of bluff bodies could be the modeling of H/h and U_t/V. Note that since U_t is provided in the calculations by Saffman & Schatzman one can have a graph for U_t/V. Now if the preferred intrinsic Strouhal number is approximately 0.28 for a vortex street generated by a fixed stationary bluff body one can relate it to the more conventional Strouhal number defined based on the characteristic length scale of the bluff body D (*e.g.* cylinder diameter) and the free stream velocity U_∞. As an example consider the Strouhal number St_D defined as

$$St_D = \frac{f_D D}{U_\infty}, \tag{12}$$

where f_D is the shedding frequency measured by an observer attached to the cylinder. An approximate value of $h/D \approx 1.2$, and $U_t/U_\infty \approx 0.16$ was reported by Tyler[24] for a fixed stationary bluff body in a flow with the free stream velocity U_∞. Therefore

$$St_D \approx \frac{1}{1.2}(1 - 0.16) = 0.7St = 0.19 \tag{13}$$

which is very close to the value reported in the literature.

6 An alternative explanation for the vortex shedding behind bluff bodies

A descriptive explanation of the nearfield wake vortex formation was offered by Gerrard[25] in the 60's. He suggested that a forming vortex draws the shear layer of the opposite sign from the other side of the wake across the wake centerline, eventually cutting off the supply of vorticity to the growing vortex. Gerrard's qualitative description of the vortex shedding process considers vortex formation solely in terms of the interaction of the main shear layer formed on the upper and lower part of the bluff body. Perry *et al.*[26] described the same process from the topology of the instantaneous-streamline patterns. The formation of a new vortex (circulating region) was characterized by the formation of a saddle point in the streamline topology.

In this section we speculate another qualitative description of the vortex shedding in the wake of bluff bodies. Instead of looking at the local dynamics of the shear layers (governed by the Euler equations) and trying to relate it to the final configuration of the resulting vortex street we take a relaxational approach (governed by *e.g.* statistical equilibrium theory. We look at the bluff body as a source for providing the flow with the invariants of motion for the Euler equations, namely the energy, impulse and circulation. From this point of view in high Reynolds number flows after the generation of the shear layers the formation of vortices is mainly an inviscid process. This assumption can be easily verified for high Reynolds number flows by comparing the viscous time scale with the relaxational time scale of the system. When this condition is satisfied the system relaxes to an almost equilibrium state, before the viscous effects alter the invariants of motion. Modification in the rate of generation of invariants of motion results in a different equilibrium state; in other words a different vortex system.

It has been experimentally observed that the aspect ratio κ in the vortex street gradually change as the viscous effects diffuse the vortices in the vortex street. In the far field of bluff bodies viscous effects cause changes in the invariants of motion (as described by the Euler equations). For example while the impulse is still conserved in a viscous fluid the kinetic energy and circulation decay. Changes in these parameters derives the system toward a different equilibrium state which itself changes continuously. Consequently, the spacing of vortices vary as the viscosity modifies the invariants of motion.

For a fixed stationary cylinder (or any other bluff body) there is a specific rate for the generation of the invariants of motion. This will result in the formation of a vortex street that is usually characterized by an aspect ratio of $\kappa = 0.28$. However any modification in the rate of generation of the invariants of motion during the formation of each circulation region might result in a different equilibrium state that might be characterized by a different spacing ratio of the vortex system. Similar to the vortex ring formation[9,10] (2D axisymmetric counterpart to the vortex shedding in 2D plane flows) it is expected that two main methods for modifying the characteristics of the resulting vortex system to be:

- changing the local speed of the shear layers
- varying the lateral spacing of the shear layers

during the formation of vortex regions. These methods substantially alter the rate of generation of the invariants of motion.

Changing the local speed of the shear layer is equivalent to changing the effective free stream velocity felt by the bluff body; consequently changing the preferred translational velocity of the resulting vortex system formed at such rate of generation of invariants of motion. Varying the lateral spacing of the shear layers could dynamically change the effective length scale of the body. Similar effect was observed in the vortex ring pinch-off process[9,10] where the dynamical variation of the piston velocity (changing the speed of the shear layers) and the exit diameter of the nozzle (changing the lateral spacing of the shear layers) resulted in the modification of the size and the shedding frequency (formation number) of the pinched-off vortex rings. See Mohseni *et al.*[10] for numerical simulation of such cases.

By appropriately forcing a periodic motion of the cylinder one can significantly modify the vortex pattern, spacing ratio of the resulting vortices, the total circulation of each vortex, and the resulting induced drag. This has been repeatedly observed over the last few decades. See the recent computations by Blackburn & Henderson[27] and the experiments by Williamson & Roshko[28] and the references in there. The preceding relaxational ideas can be used to qualitatively describe such situations. Lateral oscillation of the cylinder would dynamically modify the effective length scale of the body by changing the lateral spacing of the shear layers. Pushing the shear layer away from the wake centerline at an appropriate frequency and amplitude could result in the formation of a stronger vortex, while driving the shear layer toward the wake centerline would cut the supply of vorticity to the vortex and results in a smaller vortex. On the other hand, streamwise oscillations of the cylinder would effectively change the local speed of the shear layers. A faster shear layer can follow and feed the same sign vortex patch for a longer time and would generate a larger vortex. The appropriate amplitude and frequency of the the oscillations for the cylinder is determined by the translational velocity of the desired vortex street. Verification of these ideas and quantitative descriptions of such situations based on the relaxational ideas is the topic of a future study.

7 Conclusions

The Strouhal number St and the aspect ratio κ (the ratio of the lateral to streamwise distances in a vortex street) were studied from the frame of reference of the invariants of motion. For an observer in this frame of reference the intrinsic Strouhal number St (defined based on only the invariants of motion for 2D plane flows) and the aspect ratio κ are the same quantities.

The main invariants of motion are the energy E, impulse I, circulation Γ, and the translational velocity U_t. These invariants of motion supplemented by the geometrical invariants of motion h and l defines an intrinsic scaling for the vortex street.

The model of Saffman & Schatzman[1,23] for the Karman vortex street was used to estimate the nondimensional energy and circulation of a periodic vortex street. They consider an inviscid, incompressible, two-dimensional system consisting of vortices of finite area and uniform vorticity. For a class of vortex streets the nondimensional energy of the vortex system has a minimum around $St = \kappa \approx 0.26$, which is very close to Karman's stability criteria for point vortices. It was speculated that these cases corresponds to the vortex shedding from a fixed stationary bluff body where the rate of generation of invariants of motion are relatively constant during the formation of each vortex. By changing the relative rate of generation of invariants of motion during the formation of each circulation region one can modify the available invariants of motion in the relaxational process, resulting in a different equilibrium state with possibly different aspect ratio κ. It was suggested that relaxational point of view to the vortex shedding behind bluff bodies could provide an explanation for the effect of forced motion of bluff bodies on the characteristics of the resulting vortex street.

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