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# Vortex Shedding over a Two-Dimensional Airfoil: Where the Particles Come from

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THE ability to control vortical patterns around an airfoil is the key to altering lift and drag behavior [1]. In the field of flow control, fundamental questions regarding particle behavior must be answered. One such question is, where do the particles that make up a shed vortex pair come from and how do they form into the recognizable shape of a shed vortex pair well known to aerodynamicists? Until recently, the difficulty in pursuing this question lay in the definition of a vortex. Using vorticity as a means of defining the vortex boundary requires the choice of an arbitrary vorticity threshold. Other classical methods of defining a vortex, such as the local pressure minimum or the use of pathlines and streamlines, exhibit similar fundamental problems [2]. Alternatively, if there were a means of defining the vortex boundaries objectively, the problem would be significantly simplified. If, instead of classical definitions, the vortex boundaries were defined using stable and unstable material manifolds, there would be no need for subjective threshold values. Stable and unstable manifolds refer to two types of invariant material lines that divide the flow into regions of distinct particle mixing behavior. Stable manifolds, also referred to as repelling material lines, are responsible for the stretching of passive tracer groups normal to the manifold. Unstable manifolds (attracting material lines) stretch fluid tracer groups tangent to the manifold. In each case, the flux across a well-defined manifold boundary is negligible. For a complete explanation of stable and unstable manifolds and their properties please refer to Guckenheimer and Holmes [3].

Lagrangian coherent structures (LCS) are a tool for revealing the stable and unstable manifolds in a general flow using the time history and time future of the velocity field. These manifolds have been shown to present an objective, frame-independent definition of a vortex [4]. For a complete overview of the derivation, calculation, methods, and further references involved in generating an LCS, refer to Haller [4], Shadden et al. [5], and Cardwell and Mohseni [6]. In brief, the LCS theory uses the time future velocity field for stable manifolds and the time history velocity field for unstable manifolds to determine where the lines of hyperbolic trajectories lie in the flowfield. These lines present themselves as ridges of the finite time Lyapunov exponent (FTLE) field. For the case of a two-dimensional Eppler 387 airfoil in a low Reynolds number flow ($Re = 60,000$) at a 6 deg angle of attack, the manifolds defining the shed vortex are shown in Fig. 1a (for details of the numerical simulation, see Hall and Mohseni [7]).

In Fig. 1a, we have isolated the ridges of the FTLE values for both the stable and unstable manifolds and paired them together. Stable manifolds are represented as blue lines, whereas the unstable manifolds are represented with red lines. Focusing on the shed vortex pair, Fig. 1a shows that the structure is significantly more complicated than can be observed in classical flow maps. To illustrate this point, let us compare the vorticity map with what we find in the manifolds. Figure 1b is the same moment in time as Fig. 1a, only now we have shown the vorticity contours (black). Notice in this figure how the unstable manifolds coincide nicely with the vorticity contours in many places. The vorticity does not, however, reveal the structure formed from the stable manifolds. In particular, it does not show that the lower edge of the vortex pair is closed off or the large downstream region that is connected to the vortex pair. These are important pieces for understanding the true structure of vortex shedding. Additionally, as mentioned earlier, the vortex definition using vorticity requires an arbitrary threshold value to be selected for the vortex boundary; Fig. 1b demonstrates that only the most fortunate of guesses would result in a boundary that is roughly equivalent to the true vortex structure found using unstable manifolds. If one were to use only vorticity, it is inevitable that several parts of the vortex would be misrepresented. Instead, using the material manifolds, we are able to precisely outline the vortex boundaries and even break up the vortex into several smaller parts as defined by the intersections of the stable and unstable manifolds.

The shed vortex structure was broken into six regions based on the intersections of the stable and unstable manifolds (see Fig. 2). Each of these regions has been populated by a different-colored collection of passive, massless fluid drifter particles. When these drifters are advected backward in time, until a majority of them are upstream of the leading edge, we can observe how the structure of the vortex forms and evolves, as in Fig. 3. Notice how, even far upstream, the particles form two well-defined bubbles with distinct regions of different-colored drifters, as seen in Fig. 3a. These bubbles are separated by a stable manifold attached to the front stagnation point. Some of the particles that eventually make up the shed vortices do not

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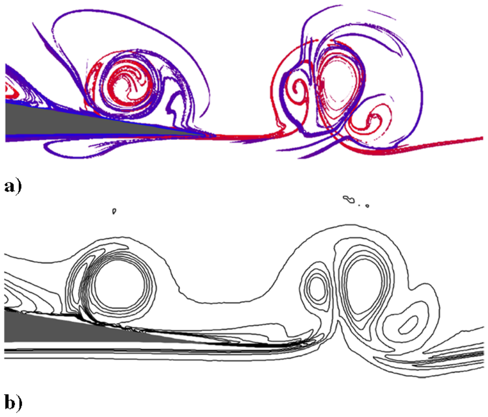

**Fig. 1** **The definition of the vortex pair once it has been shed from the top surface of the airfoil: a) stable (online version: blue) and unstable (online version: red) manifolds, and b) vorticity contour lines (black). Notice there is a rough correspondence between the unstable manifolds and the vorticity lines, but there is no similarity between the vorticity and the stable manifolds.**

**Fig. 2** **Division of the shed vortex pair for particle tracking. Regions are defined by the intersection of stable and unstable manifolds.**

originate inside of these two bubbles. Instead, these drifters are already present along the upper and lower surfaces of the airfoil. As we move forward in time, we notice that the two bubbles are split by the leading edge of the airfoil; the larger of the two moving over the top surface, whereas the smaller one advects along the bottom. For this particular vortex pair, 16.2% of the shed vortex pair originates from the lower surface bubble. Figure 3c shows that the only particles that form the vortex to come in direct contact with the airfoil's upper surface are colored black. This part of the flow forms the secondary induced vortex on the airfoil surface and eventually makes up the ejected material seen in Fig. 3f. Notice how the golden-colored particles in Fig. 3d are beginning to group together; until now, these particles have been split into two parts. In Fig. 3e, the particles from the lower surface are rolling up into the shed vortex pair. Finally, in Fig. 3f, the vortex pair is fully formed, and the full group of particles moves downstream together. The dark green and black drifters are outside of what might be traditionally described as the vortex boundary, but these two sections are included because a stable manifold encompasses these regions and connects them to the high vorticity magnitude vortex cores. This indicates that these fluid particles will remain tethered to the vortex cores for the near future.

One should notice that each particle group interacts minimally with any other particle group. This is an expected result of using material manifolds. Cross-manifold mixing is negligible, so whereas inside each lobe there is a significant mixing of particles, there is almost zero particle mixing between lobes. This can be demonstrated by taking a closer look at the vortex shedding process of particles and overlaying the manifolds. Figure 4 demonstrates how the particle groupings obey their manifold boundaries during the complicated motion of vortex shedding. Note, in particular, the behavior of the

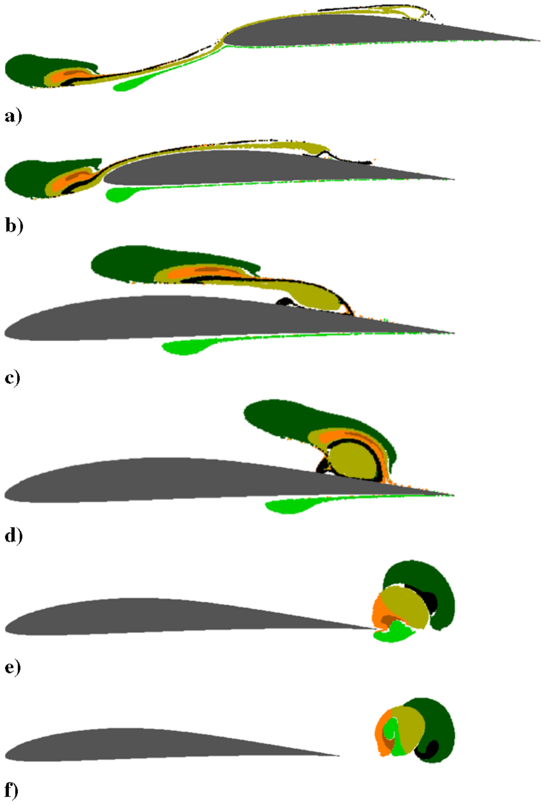

**Fig. 3** **Motion of the fluid particles that eventually make up the shed vortex pair, $\alpha = 6\ \mathrm{deg}$, $Re = 60{,}000$.**

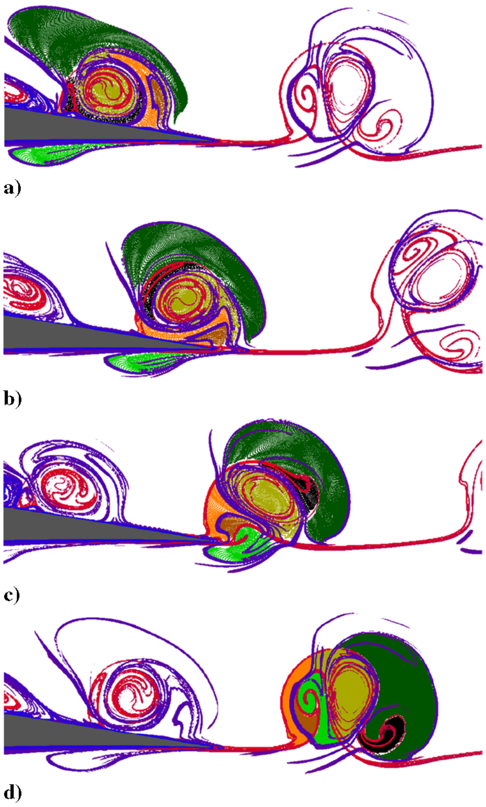

**Fig. 4** **Fluid particles with stable (online version: blue) and unstable (online version: red) manifolds showing how the vortex is divided into several regions of mixing based on the manifold intersections. $\alpha = 6\ \mathrm{deg}$, $Re = 60{,}000$.**

**Table 1 As the angle of attack of the airfoil increases, the percentage of the volume of the shed vortex originating below the airfoil decreases**

| Angle of attack | Percentage of vortex originating below airfoil |
|---|---|
| 0 | 38.8 |
| 2 | 19.0 |
| 4 | 17.7 |
| 6 | 16.2 |

unstable manifold attached to the trailing edge of the airfoil in Figs. 4b and 4c. In Fig. 4b, the manifold is intact and strong, separating the drifters from the bottom surface from those on the top. However, in Fig. 4c, this manifold has broken, allowing the vortex roll up to occur, ingesting the drifters from the bottom surface.

The various topology of vortex streets recorded in the literature can be associated with different configurations of manifolds ahead of the airfoil. This suggests that one could potentially modify the flow ahead of or even below an airfoil to obtain a different vortex configuration and, as a result, a different lift or drag characteristics. This seems counterintuitive to many recommended flow control strategies in the literature and is worth further investigation. In fact, the contribution from the lower surface increases as the angle of attack decreases. Table 1 shows that, during level flight, more than a third of the vortex forming material travels under the airfoil. A greater understanding of how fluid mixing behaves during vortex shedding results from these observations. Knowing where the particles that make up the shed vortices come from presents the opportunity to identify when these particles would be subject to flow control schemes designed to influence the size, shape, or behavior of the vortex pair.

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## References

[1] Koochesfahani, M., "Vortical Patterns in the Wake of an Oscillating Airfoil," *AIAA Journal*, Vol. 27, No. 9, 1989, pp. 1200–1205.

[2] Jeong, J., and Hussain, F., "On the Identification of a Vortex," *Journal of Fluid Mechanics*, Vol. 285, Feb. 1995, pp. 69–94. doi:10.1017/S0022112095000462

[3] Guckenheimer, J., and Holmes, P., *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer–Verlag, New York, 2002.

[4] Haller, G., "An Objective Definition of a Vortex," *Journal of Fluid Mechanics*, Vol. 525, Feb. 2005, pp. 1–26. doi:10.1017/S0022112004002526

[5] Shadden, S., Lekien, F., and Marsden, J., "Definition and Properties of Lagrangian Coherent Structures from Finite-Time Lyapunov Exponents in Two-Dimensional Aperiodic Flows," *Physica D*, Vol. 212, Nos. 3–4, Dec. 2005, pp. 271–304. doi:10.1016/j.physd.2005.10.007

[6] Cardwell, B., and Mohseni, K., "Lagrangian Coherent Structures in the Wake of an Airfoil," AIAA Paper 2007-2771, May 2007.

[7] Hall, J., and Mohseni, K., "Numerical Simulation of an Airfoil at Low Reynolds Number," AIAA Paper 2007-1269, Jan. 2007.

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