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A Dynamic Procedure for the Lagrangian Averaged Navier-Stokes-α Model of Turbulent Flows

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A dynamic procedure for the Lagrangian Averaged Navier-Stokes-α (LANS-α) equations is developed where the variation in the parameter α in the direction of anisotropy is determined in a self-consistent way from data contained in the simulation itself. In order to derive this model, the incompressible Navier-Stokes equations are Helmholtz-filtered at the grid and a test filter levels. A Germano type identity is derived by comparing the filtered subgrid scale stress terms with those given in the LANS-α equations. Assuming constant α in homogenous directions of the flow and averaging in these directions, results in a nonlinear equation for the parameter α, which determines the variation of α in the non-homogeneous directions or in time. Consequently, the parameter α is calculated during the simulation instead of a pre-defined value. In order to evaluate the applicability of the dynamic LANS-α model in anisotropic turbulence, *a priori* test of a turbulent channel flow is performed. It is found that the parameter α changes in the wall normal direction. Near a solid wall, the length scale α is seen to depend on the distance from the wall with a vanishing value at the wall. On the other hand, away from the wall, where the turbulence is more isotropic, α approaches an almost constant value. Furthermore, the behavior of the subgrid scale stresses in the near wall region is captured accurately by the dynamic LANS-α model. The dynamic LANS-α model has the potential to extend the applicability of the LANS-α equations to more complicated anisotropic flows.

I. Introduction

Turbulent flows play an important role in many areas of engineering fluid mechanics as well as atmospheric and oceanic flows. Accurate simulation of a turbulent flow requires that the energetics of the large scale energy containing eddies, dissipative small scales, and inter-scale interactions to be accounted for. In direct numerical simulations (DNS) all the involved scales are directly calculated. DNS is believed to provide the most comprehensive representation of the governing equations of fluid flows; the so-called Navier-Stokes (NS) equations. Owing to the very high Reynolds numbers encountered in most problems of interest, the disparity between the large scales and small scales, which represents the computational size of the problem, rapidly grows with the Reynolds number. Consequently, DNS can resolve only a small fraction of the turbulent activity for high Reynolds number flows.

While the direct numerical simulation of most engineering flows seems unlikely in near future, turbulence modeling could provide qualitative and in some cases quantitative measures for many applications. Large Eddy Simulations (LES) and the Reynolds Averaged Navier-Stokes Equations (RANS) are among the numerical techniques to reduce the computational intensity of turbulent calculations. In LES, the dynamics

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of the large turbulence length scales are simulated accurately and the small scales are modeled. The vast majority of contemporary LES make use of eddy-viscosity based Subgrid-Scale (SGS) models in conjunction with the spatially-averaged (filtered) Navier-Stokes Equations. In this approach, the effect of the unresolved turbulence is modeled as an effective increase in the molecular viscosity. On the other hand, RANS models are obtained by time averaging the Navier-Stokes equations. In this case most of the unsteadiness is averaged out. Consequently, the time mean quantities are calculated while the faster scale dynamics are modeled. RANS simulations are often more affordable than LES, however, their accuracy is somewhat limited in many applications.[1]

More recently, Holm, Marsden and their coworkers[2] introduced a Lagrangian averaging technique for the mean motion of ideal incompressible flows. Unlike the traditional averaging or filtering approach used for both RANS and LES, where the Navier-Stokes equations are averaged or spatially filtered, the Lagrangian averaging approach is based on averaging at the level of the variational principle. In the isotropic Lagrangian Averaged Euler-α (LAE-α) equations, fluctuations smaller than a specified scale α are averaged at the level of the flow maps.[3] Mean fluid dynamics are derived by applying an averaging procedure to the action principle of the Euler equations. Both the Euler and the Navier-Stokes equations can be derived in this manner (see Marsden & Ratiu[4] for a variational derivation of the Euler equations). The usual Reynolds Averaged Navier-Stokes (RANS) or LES equations are then obtained through the subsequent application of either a temporal or spatial average. The critical difference with the Lagrangian averaging procedure is that the Lagrangian (kinetic energy minus potential energy) is averaged *prior to the application of Hamilton principle and a closure assumption is applied at this stage.* This procedure results in either the Lagrangian averaged Euler Equations (LAE-α)[a] or the Lagrangian averaged Navier-Stokes Equations (LANS-α), depending on whether or not a random walk component is added in order to produce a true molecular diffusion term. Since the Hamilton principle is applied after the Lagrangian averaging is performed, *all the geometrical properties (e.g. invariants) of the inviscid dynamics are retained even in the presence of the model terms which arise from the closure assumption.*[2,5,6] For instance, LAE equations posses a Kelvin circulation theorem. Thus it is potentially possible to model the transfer of energy to the unresolved scales without an incorrect attenuation of quantities such as resolved circulation. This is an important distinction for many engineering and geophysical flows where the accurate prediction of circulation is highly desirable.

Numerical simulations by Chen *et al*[7] and Mohseni *et al*[8] showed the capability of the LANS-α equations in simulating isotropic homogenous turbulence. However, most engineering and geophysical flows of interest are often anisotropic. For example, due to rapid damping of turbulent fluctuations in the vicinity of a wall, the application of the isotropic LANS-α equations with a constant α is not appropriate for long term calculations. In order to capture the correct behavior in such systems the parameter α must be spatially or/and temporally varied in the direction of anisotropy,[9] *i.e.* wall normal direction. There has been some attempt (with limited success) in order to remedy this problem. A successful *dynamic* LANS-α model is yet to be formulated and tested. There are at least two approaches to anisotropy in the LANS-α equations:

(i) To derive a set of *anisotropic* LANS-α equations. See alternative derivations in.[6,10]

(ii) Use the isotropic LANS-α equations, but with a variable α to compensate for the anisotropy.

At this point much more work must be done on the anisotropic LANS-α equations before they can be applied to practical problems. The second approach listed above is what will be explored in this study.

This paper is organized as follows: The isotropic LANS-α equations and some of their main features are summarized in section II. A dynamic LANS-α approach is proposed in section III where the variation in the parameter α in the direction of anisotropy is determined in a self-consistent way from the data contained in the simulation itself. Our approach will be developed in the same spirit as the dynamic modeling procedure for conventional LES[11,12,13,14] which has achieved widespread use as very effective means of estimating model parameters as a function of space and time as the simulation progresses. The incompressible Navier-Stokes

[a] In this nomenclature, α is used to denote the filtering scale (i.e. the simulation faithfully represents motions on a scale larger than α).

equations are Helmholtz-filtered at the grid and a test filter levels. A Germano type identity is derived by comparing the filtered subgrid scale stress terms with those given in the LANS-α equations. Considering a constant value of α and averaging in the homogenous directions of the flow results in a nonlinear equation for the parameter α, which determines the variation of α in the non-homogeneous directions or time. This nonlinear equation is solved by an iterative technique. Consequently, the parameter α is calculated during the simulation instead of a fixed and pre-defined value.

Numerical techniques for simulating the dynamic LANS-α model in this study are described in section IV. In order to demonstrate the applicability of the dynamic LANS-α model of this study in anisotropic flows, *a priori* test of turbulent channel flows are performed in section V. Concluding results are presented in section VI.

II. The Isotropic LANS-α Equations

The incompressible isotropic LANS-α equations for the large scale velocity u are given by (see[2] for a derivation)

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)\, u = -\nabla p + \frac{1}{Re}\Delta u + \nabla \cdot \tau(u), \tag{1}$$

$$\nabla \cdot u = 0, \tag{2}$$

where Re is Reynolds number and $\tau(u)$ is the subgrid stress tensor defined as[3]

$$\tau(u) = -\alpha^2(1-\alpha^2\Delta)^{-1}\left[\nabla u \cdot \nabla u^T - \nabla u^T \cdot \nabla u + \nabla u \cdot \nabla u + \nabla u^T \cdot \nabla u^T\right]. \tag{3}$$

The subgrid scale stress $\tau(u)$ is in fact the momentum flux of the large scales caused by the action of smaller, unresolved scales. Here α is a constant length scale introduced during the averaging process. Note that for vanishing parameter α the NS equations will be recovered.

The LANS-α equations can be represented equivalently by

$$\frac{\partial v}{\partial t} + (u \cdot \nabla)v + v_j \nabla u_j = -\nabla P + \frac{1}{Re}\Delta v, \qquad \text{where } v_i \text{ is defined as} \qquad v = u - \alpha^2 \Delta u. \tag{4}$$

The modified pressure P in these equations is determined, as usual, from the incompressibility condition: $\nabla \cdot u = 0$ and $\nabla \cdot v = 0$.

One interpretation for the equations (1) is that they are obtained by averaging the Euler equations in Lagrangian representation over rapid fluctuations whose scale are of order α. In this respect, one can show that the Lagrangian averaged Euler equations can be regarded as geodesic equations for the H^1 metric on the volume preserving diffeomorphism group, as Arnold[15] did with the L_2 metric for the Euler equations. Note that in calculating the SGS stress $\tau(u)$ in equation (3) one needs to calculate the inverse of the Helmholtz operator $(1-\alpha^2\Delta)$, which implies the need to solve a Poisson equation. While efficient numerical treatment of the Poisson equation, or its possible elimination through rational approximation will be a focus of a future publication, we note, in passing, that the inverse of the Helmholtz operator can be expanded in α to higher orders of the Laplacian operator as shown in below

$$(1-\alpha^2\Delta)^{-1} = 1 + \alpha^2\Delta + \alpha^4\Delta^2 + \cdots \qquad \text{if} \quad \alpha\lambda_{\max} < 1 \tag{5}$$

where $\lambda_{\max}$ is the highest eigenvalue of the discretized operator Δ. It should be noted that in a periodic domain one can easily convert the equation

$$(1-\alpha^2\Delta)^{-1} v = u \tag{6}$$

to

$$\hat{v} = \frac{1}{1+\alpha^2 k^2}\hat{u} \tag{7}$$

in Fourier space. Where k is the wavenumber in Fourier space.
Now using the Taylor expansion on the right hand side one can obtain

$$\frac{1}{1+\alpha^2 k^2} = 1 - \alpha^2 k^2 + \alpha^4 k^4 - \cdots \qquad \text{if} \quad \alpha^2 k^2 < 1 \tag{8}$$

In the case of numerical discretization of Δ the convergence condition is reduced to $\alpha^2 k_{\max}^2 < 1$ where $k_{\max}$ is the highest wavemode captured in the simulation. Note that α is the cutoff scale of the Helmholtz inverse filter. Therefore, $1/\alpha$ is the highest wavemode captured accurately by the LANS-α model. The promise of the LANS-α equations is to capture the large scales ($< 1/\alpha$) accurately, while the small scales are modeled. To this end, the condition $\alpha^2 k^2 < 1$ must hold in any accurate simulation using the dynamic LANS-α equations. As a result, one can avoid solving a Poisson equation to invert the Helmholtz operator using the expansion (5).

It is interesting to note that the Lagrangian averaging technique preserves the Hamiltonian structure of the governing equations in the inviscid limit while the effects of small scales on the macroscopic features of large scale are taken into account in a conservative manner. The Hamiltonian and Lagrangian formulations of ideal fluids are both basic and useful. These formulations are part of a more general framework of *geometric mechanics*, which plays a vital role in the development of new continuum models suited for computation, as well as numerical algorithms that preserve structure at the discrete level. In recent years the geometric approach to fluid mechanics has been quite successful. Geometrical methods provide a framework for the study of nonlinear stability,[16] variational integrators,[17,18] statistical equilibrium theory,[19,20] and many other interesting topics in fluid dynamics. The Lagrangian averaged Navier-Stokes-α uses ideas from geometric mechanics and offers a theoretically and computationally attractive approach to the turbulence closure problem.

III. Derivation of a Dynamic LANS-α Model

The LANS-α equations for the large scale velocity u are given by equations (1), where $\tau(u)$ is the subgrid stress tensor defined in (3). This set of equations for α is similar to the grid filtered equation in the dynamic LES.

In analogy with the dynamic LES one can obtain an equation for the filtering length scale, α, by filtering the Navier-Stokes equations

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j}, \tag{9}$$

with the Helmholtz related filters

$$\bar{u} = (1-\alpha^2\Delta)^{-1} u, \qquad \text{grid filter}, \tag{10}$$

$$\hat{\bar{u}} = (1-\widehat{\alpha}^2\Delta)^{-1}(1-\alpha^2\Delta)^{-1} u, \qquad \text{test filter}, \tag{11}$$

to obtain

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \tag{12}$$

$$\frac{\partial \hat{\bar{u}}_i}{\partial t} + \frac{\partial \hat{\bar{u}}_i \hat{\bar{u}}_j}{\partial x_j} = -\frac{\partial \hat{\bar{p}}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \hat{\bar{u}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \tag{13}$$

where

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j,$$

$$T_{ij} = \widehat{\overline{u_i u_j}} - \hat{\bar{u}}_i \hat{\bar{u}}_j.$$

Using an idea similar to Germano identity,[11] we define

$$L_{ij} = T_{ij} - \hat{\tau}_{ij} = \widehat{\bar{u}_i \bar{u}_j} - \hat{\bar{u}}_i \hat{\bar{u}}_j, \tag{14}$$

where the subgrid scale stresses under two filtering actions can be modeled by the LANS-α subgrid term in equation (3). Therefore,

$$\tau_{ij} = \alpha^2 (1 - \alpha^2 \Delta)^{-1} M_{ij}, \tag{15}$$

$$T_{ij} = \hat{\alpha}^2 (1 - \hat{\alpha}^2 \Delta)^{-1} N_{ij}, \tag{16}$$

where

$$\begin{aligned} M_{ij} &= \frac{\partial \bar{u}_i}{\partial x_k}\frac{\partial \bar{u}_j}{\partial x_k} - \frac{\partial \bar{u}_k}{\partial x_i}\frac{\partial \bar{u}_k}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_i}, \\ N_{ij} &= \frac{\partial \hat{\bar{u}}_i}{\partial x_k}\frac{\partial \hat{\bar{u}}_j}{\partial x_k} - \frac{\partial \hat{\bar{u}}_k}{\partial x_i}\frac{\partial \hat{\bar{u}}_k}{\partial x_j} + \frac{\partial \hat{\bar{u}}_i}{\partial x_k}\frac{\partial \hat{\bar{u}}_k}{\partial x_j} + \frac{\partial \hat{\bar{u}}_j}{\partial x_k}\frac{\partial \hat{\bar{u}}_k}{\partial x_i}. \end{aligned}$$

Combining equations (14)-(16), one obtains

$$L_{ij} = \beta^2 \alpha^2 (1 - \beta^2 \alpha^2 \Delta)^{-1} N_{ij} - \alpha^2 (1 - \beta^2 \alpha^2 \Delta)^{-1} (1 - \alpha^2 \Delta)^{-1} M_{ij}, \tag{17}$$

or

$$L_{ij} = \alpha^2 (\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij}), \tag{18}$$

where $\beta = \hat{\alpha}/\alpha$. Multiplying both sides of the above equation by S_{ij}, to yield

$$L_{ij} S_{ij} = \alpha^2 (\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij}) S_{ij}. \tag{19}$$

Taking spatial averaging of both sides of the above equation in homogenous directions, one obtains

$$\alpha^2 = \frac{\langle L_{ij} S_{ij} \rangle}{\langle (\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij}) S_{ij} \rangle}, \tag{20}$$

where

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right). \tag{21}$$

The denominator in equation (20) could approach zero, where it creates a singularity. In dynamic LES, Lilly[21] used a least square approach to eliminate the singularity in Germano's model. By a similar least square approach a nonlinear equation for α could be found as

$$\alpha^2 = F(\alpha) = \frac{\langle L_{ij} (\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij}) \rangle}{\langle (\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij})(\beta^2 \hat{N}_{ij} - \hat{\bar{M}}_{ij}) \rangle}, \tag{22}$$

which does not have the singularity problem as in equation (20). This is a nonlinear equation for α. All the quantities in equation (22) can be calculated during a LANS-α simulation. Therefore, equation (22) provides a nonlinear equation for dynamically calculating the value of α during the simulation.

At this point the potential values for the free parameter β are required. Writing the grid and test filters in equations (10) and (11) in the Fourier space, one obtains

$$\breve{\bar{u}} = \frac{\breve{u}}{1 + \alpha^2 k^2}, \tag{23}$$

and

$$\breve{\hat{\bar{u}}} = \frac{\breve{u}}{(1 + \beta^2 \alpha^2 k^2)(1 + \alpha^2 k^2)} \approx \frac{\breve{u}}{1 + (\beta^2 + 1)\alpha^2 k^2} = \frac{\breve{u}}{1 + \tilde{\alpha}^2 k^2} \quad \text{as} \quad k \to \infty, \tag{24}$$

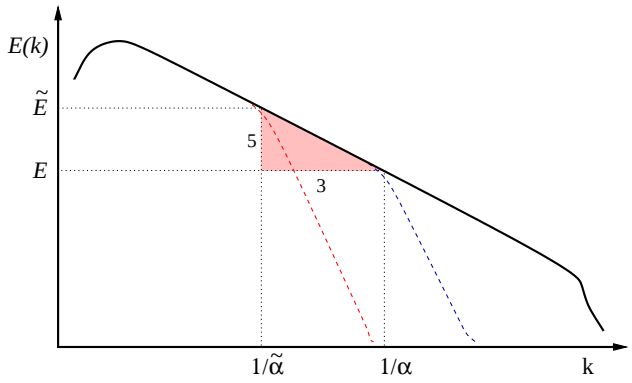


Figure 1. The positions of grid and test filter scales on the turbulent kinetic energy spectrum.

where $\check{(\cdot)}$ stands for variables in the Fourier space, k is the wavenumber, and $\tilde{\alpha}$ corresponds to filter scale for the test filter. Since $\tilde{\alpha} = \sqrt{1+\beta^2}\alpha \geq \alpha$, one can realize that as long as $\beta > 0$, the test filter have a larger filter scale than the grid filter. Figure 1 shows the relative positions of the grid filter scale α and the test filter scale $\tilde{\alpha}$ on a schematic of the energy spectrum for a high Reynolds number flow. In order to accurately model the subgrid scale stress, both the grid filter and the test filter scales must be located in the inertial sub-range of the energy spectrum. It should be pointed out that the iterative calculation required in equation (22) does not require new flow field calculations, and the iteration at each time step is carried out using the existing flow field at the same time step. Similar to the dynamic LES model, the present dynamic LANS-α model has a free parameter β, which is related to the characteristic length scale of the grid and test filters.

The dynamic α model given in equation (22) is designed to capture the length scale variations in space and time. The capability of this model in the simulation of isotropic homogenous turbulence, where α is regarded as a constant in space and changes only in time, has been tested by Zhao *et al.*[22] Aside from the isotropic homogenous turbulent flows, it is well suited for anisotropic flows such as wall bounded turbulence or mixing flow turbulence, where the turbulence length scales could change in space or in time. In cases where there are directions of homogeneity, such as the streamwise and spanwise direction in a channel flow, one can average the parameter α over the homogeneous directions. In a more general situation, we expect to replace the plane average, used in the channel flow, by an appropriate local spatial or time averaging scheme.

IV. Numerical Method

A priori test of the dynamic LANS-α procedure in a turbulent channel flow is investigated. DNS data from del Álamo and J. Jiménez[23] are employed for the *a priori* test. The computational domain in this case, normalized based on the half channel height, is spanned 8π in the streamwise and 4π in the spanwise directions. The spatial derivatives are calculated by the pesudospectral method in streamwise and spanwise directions and by the Chebychev-tau technique in the wall normal direction. Similar computational techniques have successfully been used for the DNS of channel flows by Kim *et al*[24] and Moser *et al.*[25] Grid and test filters of Helmholtz types are applied in both streamwise and spanwise directions. A grid Helmholtz type filter is applied in the wall normal direction at a fixed α value to filter the DNS data to a specific precision in this direction so that the SGS stresses get no oscillations in the near wall region. No explicit test filters are applied in the wall normal direction. α is assumed to be constant in the homogenous directions, *i.e.* the streamwise and spanwise directions, in order to solve the nonlinear equations (20) or (22). These equations

are solved by an iterative technique. The α is regarded as converged when the difference of its values at two consecutive iterations is less than 10^{-5}. The converged α values at each layer is used as an initial value for the iteration at the next grid layer. Since both the mean flow and the flow perturbations vanish at the wall, singular behavior might occur in these equations. This can be easily fixed by starting the *a priori* test a few grid points away from the wall. In actual simulation of the dynamic LANS-α equations, one can explicitly put α to zero below in the immediate vicinity of a wall when the value of α drops below a threshold.

V. Results and Discussions

Capabilities of the dynamic LANS-α model of the previous sections are examined in turbulent channel flows. *A priori* test of the dynamic LANS-α model is carried out in order to determine the accuracy of the model in predicting the SGS stresses and the energy dissipation rates in a wall bounded flow. The tests are performed on a DNS data of del Álamo and Jiménez[23] for a turbulent channel flow. The turbulence Reynolds number, based on the wall friction velocity, is $Re_\tau = 550$ and the computational grid is $1536\times257\times1536$ in the streamwise, wall normal, and spanwise directions, respectively. After dealiasing the physically relevant part of the computational domain reduces to $1024 \times 257 \times 1023$. The mean velocity profile, non-dimensionalized by the wall-shear velocity, is depicted in Figure 2(a), where a log layer from $y^+ \approx 80$ to 220 is observed. Figure 2(b) shows the turbulence intensity profiles from the wall to the middle of the channel in global coordinate which is normalized by half channel height δ. Maximum turbulence intensities in all directions are located in the wall layer.

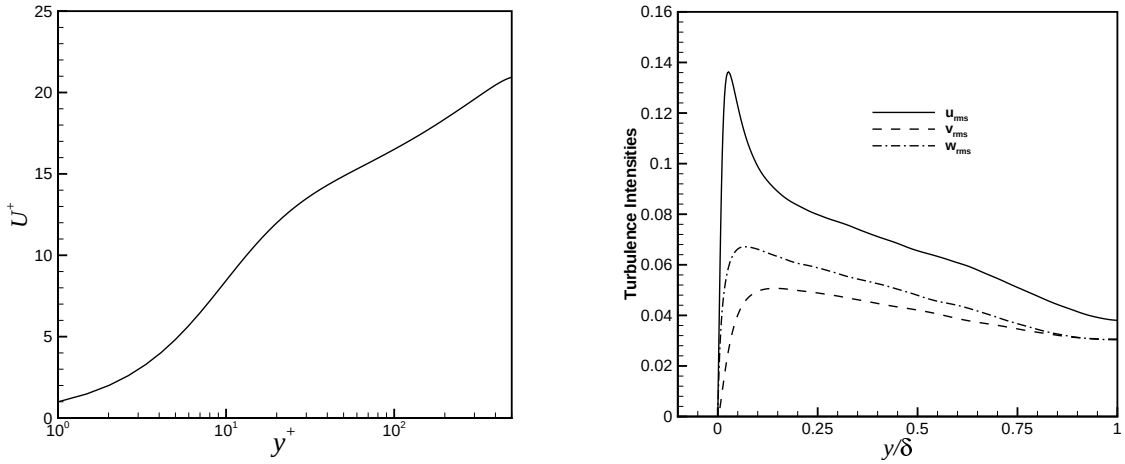
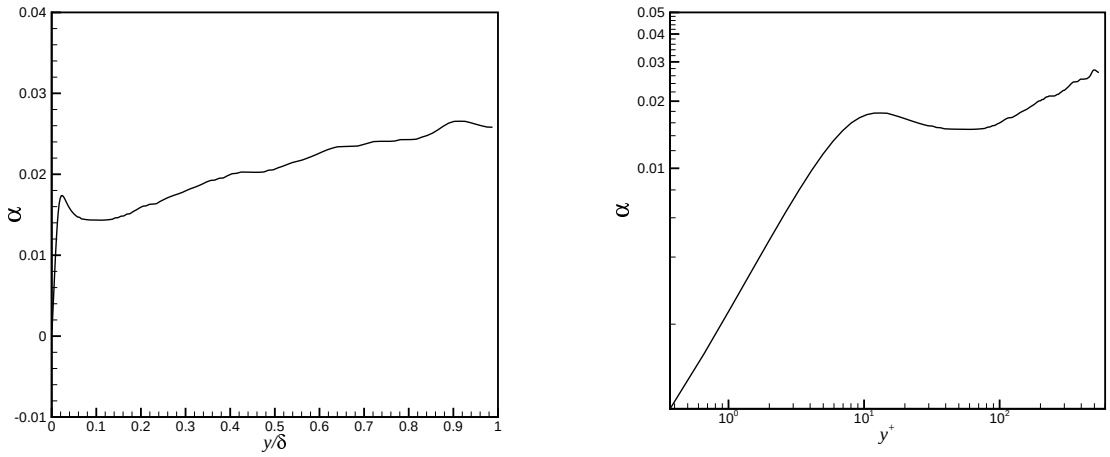


Figure 2. DNS results of a turbulent channel flow at $Re_\tau = 550$ from del Álamo and Jiménez.[23] (a) The mean velocity profile, (b) Root-mean-square velocity fluctuations in global coordinates.

Figure 3 shows the variation of α with the distance from the wall in both global and wall coordinates for $\beta = 1.0$. As demonstrated in Figure 3(b), α values experience a sharp increase in the vicinity of the wall up to $y^+ = 10$. This region of sharp increase in the value of α contains viscous sublayer. Diminishing values of α is observed as one approaches the wall. This is consistent with theoretical expectations that the NS equations ought to be recovered in the laminar layer at the wall. Away from the wall and beyond $y^+ = 10$, α shows slow decreasing then increasing across the channel. It is known that the SGS stresses will essentially approach zero in the middle of the channel, therefore, the small variation of α value in the middle of the channel has little influence on the SGS stresses. One can argue that the dynamic LANS-α equations in this

case divides the flow into two distinct regions: a near wall region that includes the viscous sublayer where α is a function of the distance from the wall, and a constant α region which includes the buffer layer, log layer and the outer layer. In the near wall region α keeps an almost log relation with the distance from the wall in wall units. In summary, one can argue that in wall bounded flows, the isotropic LANS-α calculations could be used with a constant α beyond $y^+ = 10$ and with a logarithmic relation in the near wall region. This projection requires further investigation in LANS-α calculations.



(a) (b)

Figure 3. Variation of α with distance from the wall in (a) global, and (b) wall units.

Similar to the dynamic LES, one expects that the accuracy of the dynamic LANS-α model to depend on its capability of accurately modeling the SGS stresses. The modeled and the exact SGS stresses in this flow are shown in Figure 4 for the shear stress component $< \tau_{12} >$, where $< \cdot >$ stands for averaging in streamwise and spanwise directions. The general trend of the SGS stresses are captured in the dynamic LANS-α model without any ad hoc damping function even though the dynamic model underestimates the shear component of the SGS stresses. The SGS stresses vanish at the wall and in the middle of the channel with a maximum value within the wall layer. The exact and modeled dissipation $< \varepsilon_{sgs} >$ are compared in Figure 5. It is shown that the dissipation is captured accurately by the dynamic LANS-α model in the near wall region.

VI. Conclusions

A dynamic LANS-α model is proposed where the variation in the parameter α in the direction of anisotropy is determined in a self-consistent way from data contained in the simulation itself. The model results in a nonlinear equation for α.

A priori test of the dynamic LANS-α model in a channel flow is carried out, where good agreement between the dynamic LANS-α predictions and the DNS data is observed. The parameter α is found to rapidly change in the wall normal direction in the vicinity of the wall. Near the solid wall, the length scale α shows a logarithmic dependence on the wall normal direction in wall units. Away from the wall, and in the middle of the channel, α approaches an essentially constant value. As a result, the turbulent flow is divided into two regions: a constant α region away from the wall and a near wall region. In the near wall region, α keeps an almost logarithmic relation with the distance from the wall. Consequently, one can argue that in wall bounded flows, the isotropic LANS-α calculations could be used with a constant α beyond $y^+ = 10$

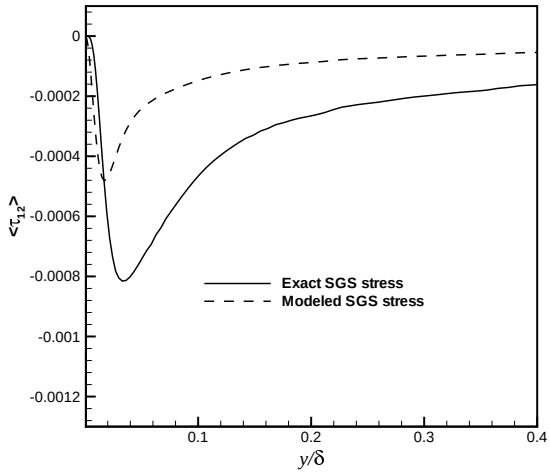


Figure 4. The averaged subgrid scale shear stress $< \tau_{12} >$ in global units.

and with a logarithmic relation in the near region. These results indicates a promising application of the dynamic LANS-α model in wall bounded turbulent flow simulations.

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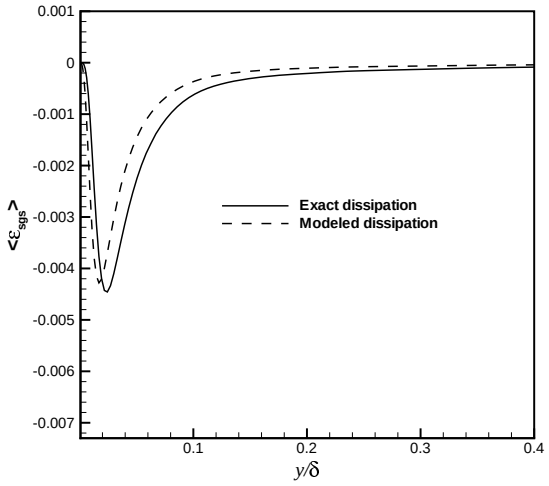

Figure 5. The averaged dissipation in global units.

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