many people since then, including Chetayev and Merkin, although the name was coined only recently (see [1]). However, the extension to the infinite dimensional case is much more subtle and this is the main point of the poster. Besides detailed analytical estimates for the PDE's involved, the tools that were useful in carrying this out for the baroclinic instability include Arnold's nonlinear stability method as well as the work of Yudovich on the linearized stability instability problem (see also the extended abstract of Friedlander and Shvydkoy).

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Isotropic LANS-α Equations for Anisotropic Turbulent Flow Simulations

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Turbulent flows play an important role in many areas of atmospheric and oceanic flows as well as engineering fluid mechanics. Accurate simulation of a turbulent flow requires that the energetics of the large scale energy containing eddies, dissipative small scales, and inter-scale interactions to be accounted for. While the direct numerical simulation (DNS) of most geophysical flows seems unlikely in near future, turbulence modeling could provide qualitative and in some cases quantitative measures for many applications. Large Eddy Simulations (LES) and the Reynolds Averaged Navier-Stokes Equations (RANS) are among the numerical techniques to reduce the computational intensity of turbulent calculations. In LES, the dynamics of the large turbulence length scales are simulated accurately and the small scales are modeled. On the other hand, RANS models are obtained by time averaging the Navier-Stokes equations. In this case most of the unsteadiness is averaged out. Consequently, the time mean quantities are calculated while the faster scale dynamics are modeled.

More recently, Holm, Marsden and their coworkers [4] introduced a Lagrangian averaging technique for the mean motion of ideal incompressible flows. Unlike the traditional averaging or filtering approach used for both RANS and LES, where the Navier-Stokes equations are averaged or spatially filtered, the Lagrangian averaging approach is based on averaging at the level of the variational principle. In the isotropic Lagrangian Averaged Euler-α (LAE-α) equations, fluctuations smaller than a specified scale α are averaged at the level of the flow maps. Mean fluid dynamics are derived by applying an averaging procedure to the action principle of the Euler equations. Both the Euler and the Navier-Stokes equations can be derived in this manner. The usual Reynolds Averaged Navier-Stokes (RANS) or LES equations are then obtained through the subsequent application of either a temporal or spatial average. The critical difference with the Lagrangian averaging procedure is that the Lagrangian (kinetic energy minus potential energy) is averaged *prior to the application of Hamilton principle and a closure assumption is applied at this stage.* This procedure results in either the Lagrangian averaged Euler Equations (LAE-α) or the Lagrangian averaged Navier-Stokes Equations (LANS-α), depending on whether or not a random walk component is added in order to produce a true molecular diffusion term. Since the Hamilton principle is applied after the Lagrangian averaging is performed, all the geometrical properties (e.g., invariants) of the inviscid dynamics are retained even in the presence of the model terms which arise from the closure assumption [4]. For instance, LAE equations posses a Kelvin circulation theorem. Thus it is potentially possible to model the transfer of energy to the unresolved scales without an incorrect attenuation of quantities such as resolved circulation. This is an important distinction for many engineering and geophysical flows where the accurate prediction of circulation is highly desirable.

However, most geophysical flows of interest are often anisotropic. For example, due to rapid damping of turbulent fluctuations in the vicinity of a wall, the application of the isotropic LANS-α equations with a constant α is not appropriate for long term calculations. In order to capture the correct behavior in such systems the parameter α must be spatially or/and temporally varied in the direction of anisotropy [2], i.e., wall normal direction. There has been some attempt (with limited success) in order to remedy this problem. There are at least two approaches to anisotropy in the LANS-α equations:

(i) To derive a set of *anisotropic* LANS-α equations. See alternative derivations in [3, 5].

(ii) Use the isotropic LANS-α equations, but with a variable α to compensate for the anisotropy.

At this point much more work must be done on the anisotropic LANS-α equations before they can be applied to practical problems. The second approach listed above is what will be explored in this study.

In this talk a *dynamic* procedure for the Lagrangian Averaged Navier-Stokes-α (LANS-α) equations is developed where the variation in the parameter α in the direction of anisotropy is determined in a self-consistent way from data contained in the simulation itself. The dynamic model is initially tested in forced and decaying isotropic turbulent flows where α is constant in space but it is allowed to vary in time. In order to evaluate the applicability of the dynamic LANS-α model in anisotropic turbulence, *a priori* test of the dynamic LANS-α in channel flows is performed at various Taylor Reynolds numbers between 180 and 550 based on the wall friction velocity to find the variation of α in the wall-normal direction. It is found that in the wall region the parameter α rapidly increases away from the wall and saturates to an almost constant value in the outer region. An appropriate scaling for α is also identified. As a result, the isotropic LANS-α equations can now be easily used in anisotropic wall bounded flows with a universally damped α. Current numerical experiments exhibit a promising application of the isotropic LANS-α model for anisotropic flows in complex geometries. For more references and details see Zhao and Mohseni [7, 6] and Bhat et.al., [1].

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