Electron vortices in semiconductors devices[a)]

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The hydrodynamic model of electron transport in semiconductors is analyzed and, in analogy with vortices in fluid mechanics, the curl of electron velocity is defined as electron vorticity, and the transport equation for the electron vorticity is derived. Aside from the classical hydrodynamic sources of vorticity, collision terms in the continuity and momentum equations are identified as sources and sinks of electron vorticity. Similar to three-dimensional fluid flows there is a vortex stretching term in the vorticity equation. This term could be responsible for the possible cascade of electron kinetic energy to small scales and formation of chaotic turbulent electron transport regimes. A scale analysis of the electron vorticity equation is performed and the relative order of magnitude of each sources of vorticity is found. This analysis and the calculation of electron mean-free-path due to electron–electron and electron–phonon scatterings characterize a transport regime with significant electron vorticity effects. Furthermore, conditions for observation of electron vortices in semiconductor devices are predicted. © *2005 American Institute of Physics*.
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I. INTRODUCTION

A dramatic decrease in size and increase in the speed of electronic devices has been observed in recent years, and the number of transistors in microelectronics has increased with an exponential pace. While current trends may continue for a while, inevitable road blocks loom. It is not clear how long this exponential path can be extended. More radical technologies and exploring new regimes of electron transport are needed in the hope of leapfrogging some of these road blocks and exploring new phenomena and designing new devices.

Electron transport in two-dimensional electron gas (2DEG) exists in three regimes: ballistic, quasi-diffusive, and diffusive. The distinction between these three regimes is defined by the relative magnitude of electron–electron scattering length, electron–phonon scattering length, and the size of the device. The diffusive transport regime for 2DEG has found application in high electron mobility transistors. There has been extensive research to include quantum mechanical effects[1,2] where the wave nature of electrons plays an important role in the device operation. To this end the ballistic transport regime has been studied extensively with the observation of conductance quantization, quantum Hall effect, and fractional quantum Hall effect. The intermediate regime of the quasi-diffusive transport has been the focus of less attention. The quasi-diffusive transport effects in 2DEG have included shallow water analogy and terahertz sources.[3–6] What distinguishes quasi-diffusive regime is that the electron temperature is high enough so that many energy levels are occupied and there is no conductance quantization or coherent electronic effects. This regime is beyond the Landauer–Buttiker formalism. In addition, the electron density is high enough for the electron–electron scattering distance to be the shortest length scale in the system. Because of the high temperatures, one can use the Hartree approximation and the relevant equations to describe electron motion are the Boltzmann transport equation and the Poisson equation.

The transport of electrons and phonons can be described by three distinct classes of physical models:

- Monte Carlo models,
- hydrodynamic models,
- drift diffusion models.

See also Table I. At the most basic level, the individual particle–particle interactions are simulated for a representative subset of the particles using the Monte Carlo method. In Monte Carlo technique the trajectory of a statistically significant number of particles in momentum and physical space are simulated (e.g., Shur[7]). There has been a considerable amount of research on the Monte Carlo methods[8,9] to consider effects such as velocity overshoot and improved modeling of heat generation in devices.[10,11] This approach is easily extendible to time scales, as short as femtoseconds and length scales down to a few nanometers. In most cases a

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TABLE I. Multiple scale electron transport in doped semiconductors. L=Characteristic length scale of film/device, $l_{e\text{-}ph}$=Average mean free path due to electron/phonon interaction, $l_{e\text{-}e}$=Average mean free path due to electron/electron interaction, λ=Wavelength of electrons. Typically $\lambda \sim l_{e\text{-}e} \ll l_{e\text{-}ph}$ in the case of moderately doped semiconductors.

	$L \ll l_{e-ph}$			$L \sim l_{e-ph}$	$L \gg l_{e-ph}$
	$L < \lambda$	$L < l_{e-e}$	$L \gg l_{e-e}$		
Transport regime	Quantum	Ballistic	Fluid	Fluid	Diffusive
Scattering	Rare	Rare	e–e (Many), e–ph (Few)		Many
Governing transport model	Wave, Schrodinger/ Green's function	Monte Carlo	Quantum Hydrodynamic / Hydrodynamic		Drift/diffusion
Applications	Quantum wells superlattices	Ballistic Transistors	**Not much explored**	**Not much explored**	Current ICs

detailed simulation of all of these particles is computationally unfeasible. One approach is to use distribution functions. To this end, the evolution of the distribution function in space and time is governed by the Boltzmann transport equation (BTE). The Boltzmann transport equation is essentially a particle number balance equation. Solving the BTE requires a complete calculation of the electron distribution function. However, a more practical approach is to solve moments of the Boltzmann transport equation that enforce the conservation of charge, energy, and momentum without requiring detailed knowledge of the particle distribution functions. This is the hydrodynamic model of electron transport.[12,13] Apart from the lower computational costs of these models (cf. Ref. 14), their similarity to the flow of compressible fluids provide an almost unlimited supply of theoretical and computational tools. This approach is especially important in fluid systems where the particle flow often has structure such as boundary layers and vortices. This sort of structure has recently been investigated in phonon and electron systems.[15,16] Finally, at the longest time and length scales the flow of particles can be described by the drift-diffusion equations, which are currently used for the design of transistors, lasers, and integrated circuits. These equations assume quasi-thermal distributions. This approach is the most widely used semiconductor device simulation tool and it is based on a coupled solution of the carrier drift-diffusion equations and the Poisson equation. The drift-diffusion equations can be derived from the Boltzmann transport equations by taking the first two moments and making simplifying assumptions. The main advantages of the drift-diffusion equations are their simplicity and that their numerical simulations are fast. However, the drift-diffusion equations neglect nonstationary transport effects and energy dependent phenomena. These effects are usually negligible in large- and low-power devices. However, such effects are of increasing importance in state-of-the-art submicron feature length semiconductor devices.

In this paper we will analyze the problem of electron transport from a hydrodynamic point of view. To this end, it is curious why electron flows in semiconductors have not shown the same rich spatial or temporal structures as in conventional fluid flows (viscosity, vortices, laminar flow, turbulence, Reynolds number effects, etc.). While the governing equations in conventional fluid dynamics are the Navier–Stokes equations, in semiconductors one often employs the drift-diffusion/Boltzmann transport equations (of course, quantum mechanics is used at very small scale). This investigation is aimed at a better understanding of the analogy and differences between electron and conventional fluid flows. Both the Navier–Stokes and the drift-diffusion equations can be derived from the moments of the Boltzmann transport equation. As discussed in the following sections, under some conditions, electron flow is dominated by the applied electric field and lattice scattering. In this case the drift-diffusion can accurately describe electron motion in a crystal. However, as device dimensions shrink below electron–phonon scattering length, ballistic effects become important and drift-diffusion is not valid anymore. Monte Carlo simulations are typically used to study device characteristics in the ballistic regime. Since Monte Carlo simulations are computationally intensive, one cannot easily simulate complicated electron flow patterns. In this study we focus on the hydrodynamic electron flow structures in semiconductors. In analogy with fluid vortices, the curl of electron translational velocity is defined as the *electron vorticity*. An electron vorticity equation is also derived and used to investigate many aspects of electron flow structures. We found that in a certain range of electron concentrations and temperatures, electron vorticity could be observed. This new regime of electron transport could be used in the design of novel devices. The current investigation is focused on electron vorticity and its associated transport; see also Mohseni *et al.*[17,18] Recently, numerical evidence for quantum vortices in semiconductor devices was reported in

electron waveguide structures[19] and dopant induced vorticity in quantum devices.[20,21]

This manuscript is organized as follows: In the next section we consider the governing equations for electron transport in semiconductor devices. We systematically derive the electron vorticity transport equation in Sec. III where similarities and differences with the vorticity equation in fluids are clarified. In general, turbulent flows exhibit high levels of fluctuating vorticity, and the vorticity equation plays an important rule in the study of turbulence. In Sec. IV we perform a scale and order-of-magnitude analysis on the electron vorticity equation to obtain the relative importance of each source term in various transport regimes. Specific example of transport in 2D electron gas in GaAs is studied in detail and range of carrier concentration, temperature and device dimension for which electron vorticity can be observed is given. Concluding remarks are provided in Sec. V.

II. GOVERNING EQUATIONS

The Boltzmann transport equation for electrons moving with the group velocity $\mathbf{u}$ in an electric field $\mathbf{E}$ can be represented as

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \nabla_{\mathbf{x}} f - \frac{e}{m} \mathbf{E} \cdot \nabla_{\mathbf{u}} f = C, \tag{1}$$

where e is the electron charge, m the effective electron mass, C the Collision term, $f(\mathbf{x},\mathbf{u},t)$ the distribution function for the electrons, $\mathbf{x}$ the space variable, and t is time.

The first three moments of the Boltzmann transport equation (1) in the velocity space are the balance equations for the flux of electron, momentum, and energy. These equations are represented as follows:[22]

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = C_n, \tag{2}$$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}(\nabla \cdot \mathbf{p}) + (\mathbf{p} \cdot \nabla)\mathbf{v} = -en\mathbf{E} - \nabla \cdot \mathbf{P} + \mathbf{C_p}, \tag{3}$$

$$m\frac{\partial}{\partial t}\left(n\left[\frac{1}{2}|\mathbf{v}|^2 + e_I\right]\right) + m\nabla \cdot \left(\mathbf{v}n\left[\frac{1}{2}|\mathbf{v}|^2 + e_I\right]\right) + \nabla \cdot (\mathbf{v}\mathbf{P}) = -en\mathbf{v} \cdot \mathbf{E} - \nabla \cdot \mathbf{q} + C_W. \tag{4}$$

Here, n is the electron concentration, $\mathbf{v}$ is the translational velocity, $\mathbf{p}$ is the momentum density $mn\mathbf{v}$, $\mathbf{P}$ is the pressure tensor, $\mathbf{q}$ is the heat flux, e_I is the internal energy, and C_n, $\mathbf{C_p}$, and C_W represent moments of C, i.e., moments of the collision terms. These equations are supplemented by the Poisson equation (in the quasi-electrostatic limit) for the electric potential ϕ

$$\mathbf{E} = -\nabla\phi, \tag{5}$$

$$\nabla \cdot (\epsilon \nabla \phi) = -\sum e_i n_i - k_1, \tag{6}$$

where k_1:=doping concentration and ϵ:=dielectric constant. For devices in which the quasi-electrostatic approximation is not accurate enough one needs to use the full Maxwell's equations (see e.g., Ref. 23).

While there are many similarities between the compressible fluid flows and the electron transport in semiconducting materials the applied electric force to charged particles (electrons) introduces some fundamental differences with fluid flows that are summarized below:

- Electric fields are the deriving forces in electron transport and are missing in most fluid flows. Note that electric fields, E, are curl free, i.e., $\nabla \times E = 0$. Therefore E does not explicitly appear in the electron vorticity transport equation (see the next section).
- There is no lattice scattering (phonons) in fluid flows. In electron transport the lattice scattering tends to destroy structures such as electron vortices (see the next section).
- There is no electron recombination in fluid flows.

These differences introduce new characteristics that are investigated in the following sections.

III. ELECTRON VORTICITY TRANSPORT EQUATION

In analogy with fluid vortices, the curl of electron translational velocity is defined as the *electron vorticity*. Here we follow the fluid dynamics approach (e.g., see Batchelor[24]) to derive an equation for electron vorticity transport. Using the continuity equation we can write the curl of the acceleration term as

$$\nabla \times \frac{D\mathbf{v}}{Dt} = n\frac{D}{Dt}\left(\frac{\vec{\omega}}{n}\right) + \frac{C_n}{n}\vec{\omega} - (\vec{\omega} \cdot \nabla)\mathbf{v}, \tag{7}$$

where $\vec{\omega}$ is the electron vorticity vector $\nabla \times \mathbf{v}$, and

$$\frac{D}{Dt} := \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla, \tag{8}$$

is the material derivative. This is, in fact, the modified Beltrami vorticity equation[25] that includes sources due to electron generation and recombination. Noting that $\nabla \times E = 0$, and using Eqs. (7), one can calculate the curl of the momentum Eq. (3) to obtain

$$n\frac{D}{Dt}\left(\frac{\vec{\omega}}{n}\right) - (\vec{\omega} \cdot \nabla)\mathbf{v} = -\nabla \times \left(\frac{1}{mn}\nabla \cdot \mathbf{P}\right) + \nabla \times \left(\frac{1}{mn}(\mathbf{C_p} - mC_n\mathbf{v})\right) - \frac{C_n}{n}\vec{\omega}. \tag{9}$$

In the relaxation time approximation, the collision terms are modeled as

$$C_n = -R, \tag{10}$$

$$\mathbf{C_p} = -\frac{\mathbf{p}}{\tau_p}, \tag{11}$$

$$C_W = -\frac{W - W_0}{\tau_w}, \tag{12}$$

where R is the recombination rate and τ_p and τ_w are the momentum and energy relaxation times, respectively.

Now we need a constitutive law (moment closure) for the pressure tensor $\mathbf{P}$.[26,27] For simplicity, we consider an in-

viscid model, where we assume that the pressure tensor can be represented in terms of the effective carrier temperature T by an ideal gas law relationship

$$\mathbf{P} = nkT\mathbf{I}. \tag{13}$$

Here $\mathbf{I}$ is the identity tensor, and k is the Boltzmann constant. Therefore, the vorticity equation (9) can be written as

$$n\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\left(\frac{\vec{\omega}}{n}\right) = (\vec{\omega}\cdot\nabla)\mathbf{v} - \frac{k}{m}\nabla\left(\frac{T}{n}\right)\times\nabla n + \left(2\frac{R}{n} - \frac{1}{\tau_p}\right)\vec{\omega} + R\nabla\left(\frac{1}{n}\right)\times\mathbf{v}. \tag{14}$$

This is the electron vorticity transport equation. Note that for simplicity we derived the electron vorticity equation for an inviscid constitutive relation (13) where there is no vorticity redistribution due to diffusion. For the viscous constitutive relations for $\mathbf{P}$ a diffusion term will be added to the electron vorticity equation. The main advantage of this equation over the classical hydrodynamic models is that the electric field does not appear explicitly in the electron transport equation. This is due to the fact that electric fields are curl free.

Equation (14) shows that the ratio of the electron vorticity to the electron concentration (following an electron concentration element) can change with time due to the terms on the right-hand side of Eq. (14). There are seven terms involved in the electron vorticity transport equation (14). The two terms on the left-hand side form the material derivative of the vorticity density, $\vec{\omega}/n$. The third term represents the vortex stretching essential for turbulence. In three-dimensional flows, vortex stretching is responsible for the cascade of kinetic energy to smaller scales (energy transfer from the mean flow to the fluctuations), which is necessary for the formation of turbulent and chaotic flows. This is consistent with recent observation of chaotic electron flows in semiconductors.[28] In general, turbulence is characterized by high levels of fluctuating vorticity. These random vorticity fluctuations do not persist in two-dimensional flows, since the main vorticity-maintenance mechanism, vortex stretching, is absent in these flows. The fourth term is similar to the baroclinic generation of vorticity in fluid mechanics and is due to the interaction of the principal part of the pressure tensor $\mathbf{P}$ and the density field n. While the principal part of the pressure tensor $\mathbf{P}$ acts on the center of a differential element of the electron flow, the electric field acts on a different point inside the element if the electron density is not uniformly distributed over the element (see Fig. 1). The net force acting on these two points apply a torque on the element, and is the essential mechanism for the baroclinic vorticity generation. The last three terms in Eq. (14) are due to vorticity generation through the collision terms in the Eqs. (2) and (3).

It is clear that in various regimes of electron transport, different terms in the electron vorticity equation are dominant. In large systems and under normal field conditions the vorticity sink term, due to the interaction of vortices with the lattice, damps out most of the electron vorticity generation.

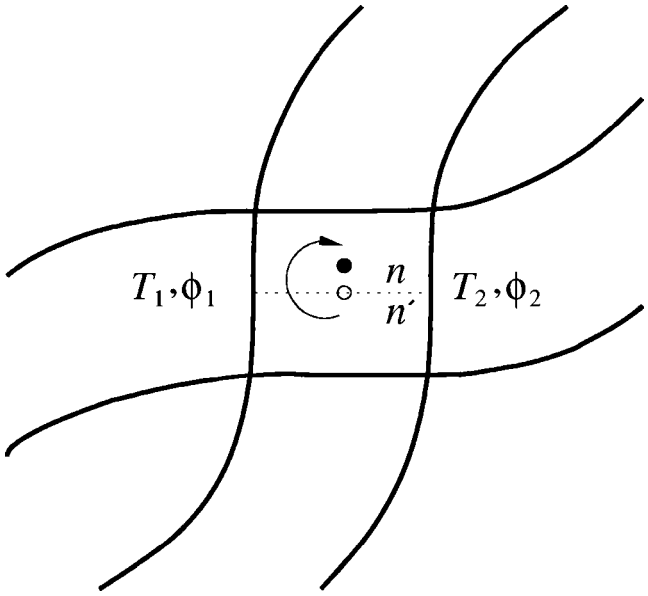

FIG. 1. Baroclinic electron vorticity generation in a differential element of electron flux with $n>n'$: The temperature gradient acts on a center of mass (○) and the gradient in electrostatic potential acts on a center of electron charge at (●). Since the two gradients act on different points within the differential element, there is a net torque on the electron fluid.

In the next section we perform an order of magnitude analysis to predict transport regimes in which electron vorticity dynamics play an important role.

IV. SCALING AND ORDER OF MAGNITUDE ANALYSIS

Since the strength of the electric field applied to the device can vary significantly, it is interesting to compare the order of each term in the vorticity equation. We assume that the characteristic scales of the problem, i.e., velocity, length, electron concentration, temperature, and electric field are given by U, L, n_0, T_0, E_0, respectively. Note that the time scale is given by $\tau = L/U$. We can now introduce nondimensional variables (assuming that the scaling is the same in each direction)

$$\mathbf{x}^* = \frac{\mathbf{x}}{L}, \quad \mathbf{v}^* = \frac{\mathbf{v}}{U}, \quad t^* = \frac{U}{L}t, \quad n^* = \frac{n}{n_0}, \tag{15}$$

$$\vec{\omega}^* = \frac{L}{U}\omega, \quad T^* = \frac{T}{T_0}, \quad \mathbf{E}^* = \frac{\mathbf{E}}{E_0}, \quad \nabla^* = \frac{\nabla}{L}. \tag{16}$$

Now, the appropriate scaling for T_0 and E_0 should be found. The scaling for E_0 can be easily obtained from Eq. (5),

$$E_0 = \frac{\phi_0}{L_d}, \tag{17}$$

where ϕ_0 is the scaling for the electric potential applied to the device. The scaling for T_0 may be obtained from the energy Eq. (4). In doing so we assume that the order of the main driving term $en\mathbf{v}\cdot\mathbf{E}$ is the same as the convective derivative on the left-hand side. Of course, after such an assumption one should check its validity at the end of the calculations. Hence

$$T_0 = \frac{e\phi_0}{k}. \tag{18}$$

It is important to note that even though the electric field does not enter directly in the vorticity equation, it sets the scaling for electron temperature in the device. Now we can write the nondimensional vorticity equation with the appropriate scaling,

$$n^*\left(\frac{\partial}{\partial t^*} + \mathbf{v}^* \cdot \nabla^*\right)\left(\frac{\vec{\omega}^*}{n^*}\right) = (\vec{\omega}^* \cdot \nabla^*)\mathbf{v}^* - \frac{e\phi_0}{mU^2}\nabla^*\left(\frac{T^*}{n^*}\right) \times \nabla^* n^* + 2\frac{RL}{Un_0}\frac{\vec{\omega}^*}{n^*} - \frac{L}{U\tau_p}\vec{\omega}^* + \frac{RL}{Un_0}\nabla^*\left(\frac{1}{n^*}\right) \times \mathbf{v}^*. \tag{19}$$

The three nondimensional numbers that appear on the right-hand side of Eq. (19) are of fundamental importance in our analysis. The nondimensional number $e\phi_0/mU^2$ of the baroclinic term is the ratio of the absorbed energy of a free electron from the external potential ϕ_0 to the average thermal energy of electrons. One can obtain an estimate for this nondimensional number by using Eq. (3). If assuming that the order of the driving force, $(e/m)\mathbf{E}$, is the same as the acceleration term $D\mathbf{v}/Dt$, and using (17) we obtain $mU^2 \approx e\phi_0$. This implies that the baroclinic term is of the same order as the vorticity acceleration and vortex stretching terms. The nondimensional number $L/U\tau_p = \tau/\tau_p$ in front of the momentum relaxation source term is, in fact, the ratio of the transit time to the momentum relaxation time. Note that the recombination rate can be represented as $R \approx n_0/\tau_r$. Therefore, the nondimensional number $RL/Un_0 = \tau/\tau_r$ in front of the recombination term in Eq. (19) can be interpreted as the ratio of the transit time in the device to the recombination relaxation time.

In most cases one can neglect the vorticity generation by the recombination term; τ/τ_r is usually a very small number. Since the momentum relaxation term acts as a sink of electron vorticity, one expects to observe the transport of electron vortices in a regime in which this term is smaller than the other remaining source terms, i.e., it is of the order of one or less. A simple calculation of τ/τ_p for a semiconductor material with $\tau_p = 1$ ps (e.g., GaAs at low temperatures) shows that for easily achievable feature sizes (0.1–1.5 μm) and applied electric potentials (0.1–1 V), the order of magnitude of this term is around one and significant electron vorticity generation and convection can be observed (see Fig. 2). This is a new electron transport regime that has not been rigorously investigated before. In order to verify that this regime of transport falls within the range of hydrodynamic flow assumption, we calculate electron mean-free-path due to electron–electron (l_{e-e}) and electron–phonon (l_{e-ph}) scatterings.

In Fig. 3, the relevant scattering lengths are plotted as a function of electron temperature T_e: the electron–electron scattering length $l_{e-e} = v_F\langle\tau_{e-e}\rangle$, the impurity scattering length $l_I = v_F\langle\tau_i\rangle$, and the phonon scattering length $l_{ph} = v_F\langle\tau_{ph}\rangle$, where v_F is the Fermi velocity and $\langle\tau_{e-e}\rangle$, $\langle\tau_i\rangle$, and $\langle\tau_{ph}\rangle$ are

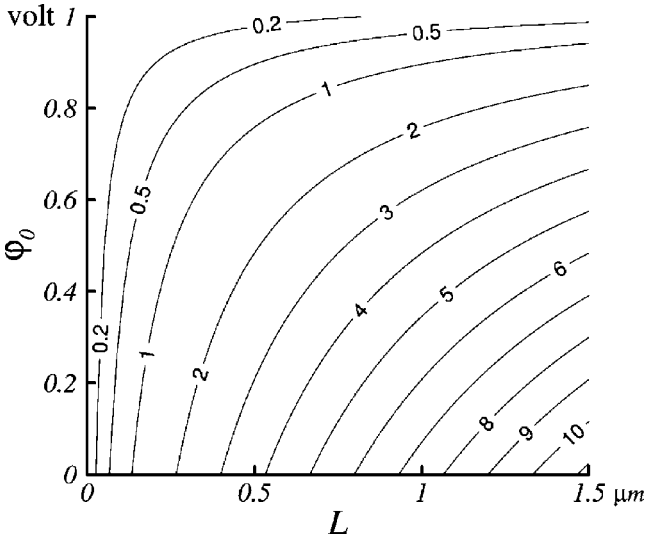

FIG. 2. Contour plot of τ/τ_p with $\tau_p = 1$ ps. For higher voltage regimes (e.g., $\phi_0 \gtrapprox 0.8$) the intervalley transfer of electrons should be considered.

the electron–electron, impurity, and phonon scattering times, respectively. In these calculations, a low temperature 2DEG mobility of $\mu(T=0) = 2\times10^6$ cm^2/Vs and density $n = 2.7\times10^{11}$/cm^2 was assumed. The calculation of the electron–phonon scattering is based on Fig. 8 of Kawamura.[29] Acoustic phonon deformation potential and piezoelectric scattering have been considered with density dependent screening accounted for by the random-phase approximation (RPA). As shown in Kawamura,[29] the calculated scattering rates agree well with measured mobilities over the range of temperatures and densities considered here. The electron–electron mean free path is determined by the expression used by Gurzhi.[4,5] Again the estimated electron–electron mean free path has been carefully verified with experimental results in the ballistic regime.[30] It is evident from Fig. 3 that at $T_e = 20$ K, $l_{e-e} \approx 0.5$ μm is much smaller than l_m and if a wire with

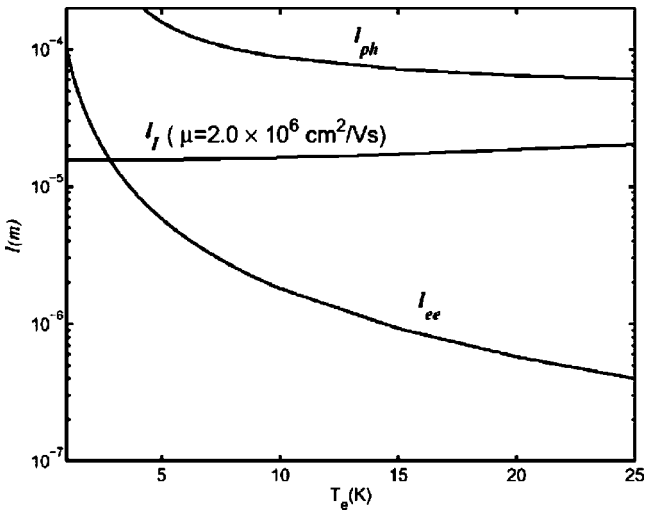

FIG. 3. The electron-impurity (l_I), electron–phonon (l_{ph}), and electron–electron scattering lengths for a 2DEG with the mobility $\mu(T=0) = 2\times10^6$ cm^2/V s and density $n = 2.7\times10^{11}$/cm^2. The lattice temperature T_L is fixed at 1 K and the electron temperate T_e is varied.

diffuse boundary scattering is fabricated with a width of $\approx 5\ \mu$m, then a Poiseulle like regime as first predicted by Ref. 31 might form. Since the electron–electron mean free path is of the same order as the phase-breaking length (l_{phi}), the transport regime where the electron density and temperature is such that $l_{\mathrm{e-ph}} > l_{\mathrm{e-e}} = l_{\mathrm{phi}}$ corresponds to a regime where the macroscopic transport is purely classical and is well described by the hydrodynamic equations. For a range of device dimensions between $0.1-1\ \mu$m this corresponds to densities of approximately 10^{11} cm^{-2} and temperatures between 10–40 K. At higher densities or lower temperatures the phase-breaking length ($l_{\mathrm{phi}} = l_{\mathrm{e-e}}$) becomes equal to the device dimension and a classical description fails. At higher temperatures, $l_{\mathrm{e-ph}}$ is on the order of the device dimension and there is strong damping on the evolution of any generated electron vortices. It should also be mentioned that fast electron–electron scattering destroys quantum mechanical phase coherence. This has been demonstrated in Aharanov–Bohm experiments in the temperatures of interest in this study.

For a classical gas, viscosity is given by $\mu_c = \frac{1}{2} mnv_T\lambda$, where λ is the interparticle scattering length (mean free path), m the mass of the particle, n the gas number density, and v_T the thermal velocity.[32] For an electron Fermi gas, the analogous expression for viscosity is $\mu_e \sim mnv_F l_{\mathrm{e-e}}$[32] where m is the effective mass of the electrons and v_F is the Fermi velocity. For typical devices of interest in this study one can easily estimate the Reynolds number $Re = UL/\nu$ to be of the order of 10. Similar estimate for the Reynolds number of a 2DEG was reported by Dyakonov and Shur.[33]

In summary, for submicron devices at moderately low temperature one can expect considerable electron vorticity formation. This new regime of electron transport is interesting for both a fundamental point-of-view and for applications. Many effects observed in classical fluid dynamics could be studied in the electron gas in semiconductors. In addition, the rich structure of the flow (e.g., vorticity formation and shedding) and the importance of the boundary conditions can be used in the design of novel devices.

V. CONCLUSIONS

We analyzed the hydrodynamic model of electron transport in semiconductors and in analogy to fluid mechanics the curl of electron group velocity is defined as the electron vorticity, $\vec{\omega} = \nabla \times \mathbf{v}$ (proposed in Ref. 17 and 18). An equation for electron vorticity transport is derived. It is found that in addition to the conventional stretching term and baroclinic generation of vorticity (e.g., see Ref. 24), the other sources of electron vorticity are the generation of electron vortices due to the recombination term and decay of vortices due to the momentum relaxation. To simplify our analysis, the diffusion term in the modeling of pressure tensor is neglected. This assumption is valid for low viscosity and away from the boundaries. For regions close to nonconductive boundaries, a diffusion term is needed in the right hand side of the vorticity transport equation to provide a means for the diffusion of electron vorticity, created at the boundary, into the conductive region. This can be achieved through electron–electron interaction and nondiagonal terms in the pressure tensor (viscosity). The rate of vorticity generation at the boundary is set by the boundary conditions. To obtain a complete set of equations of electron transport in semiconductors one needs to supplement the vorticity equation (14) by an equation for electron dilatation, $\nabla \cdot \mathbf{v}$. This is the topic of a future publication.

A scale analysis of the electron vorticity equation is performed and the relative order of magnitude of each source of electron vorticity is found. In most cases, electron-lattice scattering will destroy any coherent structure in electron flows. This could explain why there has been no observation of electron vortices (coherent structures) in semiconductor devices. However, our analysis predicts conditions for a new regime of electron transport in submicron to micron size devices at moderately low temperature, where significant electron vorticity generation and structures are expected. This result could guide experimentalists and numerical analysts of semiconductor devices to explore formation of electron vortices. Controlled vorticity formation and the spatial and temporal structure in electron flow can be used in the design of novel devices. This investigation is set in the regime of validity of hydrodynamic models, i.e., the characteristic length scales are so that the quantum mechanical effects can be neglected and the electron-electron scattering is fast enough so that one cannot make the independent electron approximation. In order to observe electron vortices, electron transit time in a device should be of the same order as the momentum relaxation time. In addition, the device geometry, boundary effects, material properties, and temperature should be chosen appropriately.

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